Pensieve header: Scatter and Glow in OneCo. Continues pensieve://2016-02/.

In the U(T)U(H) conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-03"]
```

I ought to be able to replace c[i] with a[c,i], and ca with aa, everywhere!

Generalities

```
DQ[is___] := (Sort[{is}] === Union[{is}]);
OQ[is___] := OrderedQ[{is}];
(* tests for non-strict ordering. Also true when {is} is {i,i}. *)
Kδ_is__ := KroneckerDelta[1, Length[Union[{is}]]];

Simp[expr_] := Expand[expr];
S[expr_] :=
  expr /. (λ_β | λ_a | λ_δβ | λ_δa | λ_c | λ_ca | λ_δaa) :> MapAt[Simp, λ, 1];

AutoCollecting[λ_] := (
   λ /: λ[0, ___] = 0;
   λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f + g], r];
   λ /: g_ * λ[f_, r___] := λ[Simp[g f], r];
  );
AutoCollecting /@ {β, a, δβ, c, δa, ca, δaa};
UU /: UU[x_] + UU[y_] := UU[x + y];
UU /: a_ * UU[x_] := UU[Expand[a x]];

γ[f_, j_, k_] := δa[f, j, k] - c[ϵ48 b_j f, k];
γa[f_, j_, k_, l_, m_] := δaa[f, j, k, l, m] - ca[ϵ48 b_j f, k, l, m];
```

Bases

```
UUBasis[T_List, H_List, f_] := Module[
    {ff, n = 0, h, t, h1, h2},
    ff := f_{++n} @@ Table[b_t, {t, T}];
    UU /@ Flatten@{
         β[ff],
         Table[{a[ff, t, h], δa[ff, t, h]}, {t, T}, {h, H}],
         δβ[ff],
         Table[c[ff, h], {h, H}],
         Table[ca[ff, h1, t, h2], {h1, H}, {t, T}, {h2, H}],
         Table[δaa[ff, T[[i]], H[[j]], T[[k]], H[[l]]],
          {k, Length@T}, {i, k}, {l, Length@H}, {j, l}]
        } /. 1_[___] → 1
  ];
UUBasis[S_List, f_] := UUBasis[S, S, f];
UUBasis[n_Integer, m_Integer, f_] := UUBasis[Range@n, Range@m, f];
UUBasis[n_Integer, f_] := UUBasis[Range@n, f];
```

δaa relations

Switch from thth to tthh indexing? (not for the moment)

```
UU[expr_] // S := UU[S[expr]]; (* Temporary fixture! *)
```

```
UU[expr_] // S := UU[S[expr //. {
       δaa[f_, i_, j_, k_, l_] /; !OQ[j, l] :↦ δaa[f, k, l, i, j],
       δaa[f_, i_, j_, k_, l_] /; !OQ[i, k] ⋀ DQ[j, l] ⋀ OQ[j, l] :↦
        δaa[f, i, l, k, j] + ca[ϵ4 b_k f, l, i, j] + ca[-ϵ4 b_i f, l, k, j] +
         ca[-ϵ4 b_k f, j, i, l] + ca[ϵ4 b_i f, j, k, l],
       δaa[f_, i_, k_, j_, k_] /; !OQ[i, j] :↦ δaa[f, j, k, i, k] +
         δa[-ϵ2 b_i f, j, k] + δa[ϵ2 b_j f, i, k]
      }]];
```

tm, hm, hts, dm

```
UU[expr_] // tm[x_, y_, z_] := S[UU[Expand[expr /. {
        a[f_, x, j_] :> a[f, z, j] + ϵ13 γ[∂b_y f, z, j],
        a[f_, y, j_] :> a[f, z, j],
        δa[f_, x | y, j_] :> δa[f, z, j],
        ca[f_, i_, x | y, j_] :> ca[f, i, z, j],
        δaa[f_, i_, j_, k_, l_] :>
         δaa[f, i // Replace[x | y → z], j, k // Replace[x | y → z], l]
       } /. b_{x|y} → b_z]]];

UU[expr_] // hm[x_, y_, z_] := S[UU[Expand[expr /. {
       a[f_, i_, x | y] :> a[f, i, z],
       c[f_, x | y] :> c[f, z],
       δa[f_, i_, x | y] :> δa[f, i, z],
       ca[f_, y, j_, x] :> ca[f, z, j, z] + ϵ12 γ[f, j, z],
       ca[f_, i_, j_, k_] :>
        ca[f, i // Replace[x | y → z], j, k // Replace[x | y → z]],
       δaa[f_, i_, y, k_, x] :> δaa[f, k, z, i, z],
       δaa[f_, i_, j_, k_, l_] :>
        δaa[f, i, j // Replace[x | y → z], k, l // Replace[x | y → z]]
      }]]];

UU[expr_] // hts[y_, x_] := S[UU[Expand[expr /. {
       a[f_, i_, j_] :> a[f, i, j] - ϵ11 Kδ_{j,y} γ[∂b_x f, i, y] -
         Kδ_{i,x} Kδ_{j,y} (ϵ5 β[f b_x] + ϵ6 c[f, y] - ϵ7 δβ[b_x ∂b_x f]),
       δa[f_, x, y] :> δa[f, x, y] - ϵ8 δβ[f b_x],
       ca[f_, i_, j_, k_] :>
        ca[f, i, j, k] + ϵ10 Kδ_{i,y} Kδ_{j,x} γ[f, x, k] + Kδ_{j,x} Kδ_{k,y} c[-ϵ9 f b_x, i],
       δaa[f_, i_, j_, k_, l_] :> δaa[f, i, j, k, l] + ϵ14 Kδ_{i,x} Kδ_{j,y} δa[-b_x f, k, l] +
         ϵ3 Kδ_{i,x} Kδ_{l,y} (-δa[b_k f, x, j] + δa[b_x f, k, j]) +
         ϵ1 Kδ_{k,x} Kδ_{j,y} (δa[b_i f, x, l] - δa[b_x f, i, l]) + ϵ15 Kδ_{k,x} Kδ_{l,y} δa[-b_x f, i, j] +
         ϵ16 Kδ_{i,x} Kδ_{j,l,y} δβ[b_x b_k f] + 2 ϵ0 Kδ_{x,i,k} Kδ_{y,j,l} δβ[b_x b_x f]
      }]]];

dm[x_, y_, z_][expr_] := expr // hts[x, y] // tm[x, y, z] // hm[x, y, z]
```

tσ, hσ, dσ on {β, a, δβ, c, δa, ca, δaa}

```
tσ[x_List, y_List][expr_] := Module[{rule = Thread[x → y]},
   expr /. b_i_ :> b_i /. rule /. {
     a[f_, i_, j_] :> a[f, i /. rule, j],
     δa[f_, i_, j_] :> δa[f, i /. rule, j],
     ca[f_, i_, j_, k_] :> ca[f, i, j /. rule, k],
     δaa[f_, i_, j_, k_, l_] :> δaa[f, i /. rule, j, k /. rule, l]
    }
  ];
tσ[x_, y_][expr_] := tσ[{x}, {y}][expr];
hσ[x_List, y_List][expr_] := Module[{rule = Thread[x → y]},
   S[expr /. {
      a[f_, i_, j_] :> a[f, i, j /. rule],
      c[f_, i_] :> c[f, i /. rule],
      δa[f_, i_, j_] :> δa[f, i, j /. rule],
      ca[f_, i_, j_, k_] :> ca[f, i /. rule, j, k /. rule],
      δaa[f_, i_, j_, k_, l_] :> δaa[f, i, j /. rule, k, l /. rule]
     }]
  ];
hσ[x_, y_][expr_] := hσ[{x}, {y}][expr];
dσ[x_, y_][expr_] := expr // tσ[x, y] // hσ[x, y];
```

tb, hb, thb, htb, db, bb on {β, a, δβ, c, δa, ca, δaa}

```
tb[x_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
          p[0, _] → 0, p[_, 0] → 0,
          p[_β | _δβ | _c | _δa | _ca | _δaa, _β | _δβ | _c | _δa | _ca | _δaa] → 0,
          p[u_β | u_δβ | u_c | u_δa | u_ca | u_δaa, v_a] :> -p[v, u]
         } /. {
         p[a[f_, x, j_], u_] :> (u /. {
             β[g_] :> ϵ39 γ[f ∂b_x g, x, j],
             a[g_, k_, l_] :> ϵ40 γa[f ∂b_x g, x, j, k, l] + Kδ_x,k (-γa[ϵ41 g ∂b_k f, k,
                    l, x, j] + ca[ϵ17 f g, l, x, j] - ca[ϵ17 f g, j, k, l]),
             _ → 0
            }),
         p[a[f_, j_, k_], a[g_, x, l_]] :> -γa[ϵ42 g ∂b_x f, x, l, j, k],
         p[_, _] → 0
        }]]]];
```

```
hb[y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
          p[0, _] → 0, p[_, 0] → 0,
          p[_β | _δβ, _] → 0,
          p[_, _β | _δβ] → 0,
          p[_c | _δa | _ca | _δaa, _c | _δa | _ca | _δaa] → 0,
          p[u_c | u_δa | u_ca | u_δaa, v_a] :↦ -p[v, u]
         } /. {
         p[a[f_, i_, y], u_] :↦ (u /. {
              a[g_, j_, k_] :↦ ϵ18 Kδ_{y,k} (a[b_j f g, i, y] - a[b_i f g, j, k]),
              c[g_, j_] :↦ ϵ19 Kδ_{y,j} γ[f g, i, j],
              δa[g_, j_, k_] :↦ ϵ20 Kδ_{y,k} (δa[b_j f g, i, y] - δa[b_i f g, j, k]),
              ca[g_, j_, k_, l_] :↦ Kδ_{y,j} γa[ϵ21 f g, i, j, k, l] +
                Kδ_{y,l} (ca[ϵ22 b_k f g, j, i, y] - ca[ϵ22 b_i f g, j, k, l]),
              δaa[g_, j_, k_, l_, m_] :↦ ϵ23 Kδ_{y,k} (δaa[b_j f g, i, y, l, m] - δaa[b_i f g, j,
                    k, l, m]) + ϵ24 Kδ_{y,m} (δaa[b_l f g, j, k, i, y] - δaa[b_i f g, j, k, l, m])
             }),
          _p → 0
         }]]]];
```

```
thb[x_, y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
        p[0, _] → 0, p[_, 0] → 0,
        p[_β | _δβ | _c | _δa | _ca | _δaa, _β | _δβ | _c | _δa | _ca | _δaa] → 0,
        p[_a, _β | _δβ] → 0,
        p[β[f_], a[g_, i_, j_]] :↦ Kδ_{y,j} γ[ϵ43 g ∂_{b_x} f, i, y],
        p[a[f_, i_, j_], a[g_, k_, l_]] :↦ Kδ_{y,l} (
            γa[ϵ44 g ∂_{b_x} f, k, l, i, j] + Kδ_{x,i} (
                γ[-ϵ45 b_k g ∂_{b_x} f, i, j] + δa[ϵ46 b_k g ∂_{b_x} f,
                  i, j] - δa[ϵ47 b_i g ∂_{b_x} f, k, j] - a[ϵ25 b_k f g, i, j] + a[
                  ϵ25 b_i f g, k, j] + ca[ϵ26 f g, j, k, l] - ca[ϵ26 f g, l, k, j]
              )
          ),
        p[a[f_, i_, j_], c[g_, k_]] :↦ -ϵ27 Kδ_{i,x} Kδ_{k,y} γ[f g, i, j],
        p[a[f_, i_, j_], δa[g_, k_, l_]] :↦
         ϵ28 Kδ_{x,i} Kδ_{y,l} (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
        p[a[f_, i_, j_], ca[g_, k_, l_, m_]] :↦ Kδ_{x,i} (
            -ϵ29 Kδ_{y,k} γa[f g, i, j, l, m] + ϵ30 Kδ_{y,m}
              (-ca[b_l f g, k, i, j] + ca[b_i f g, k, l, j]) - ϵ31 Kδ_{y,k,m} γ[b_l f g, x, j]
          ),
        p[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] :↦ Kδ_{x,i} (
            ϵ32 Kδ_{y,l} (-δaa[b_k f g, i, j, m, n] + δaa[b_i f g, k, j, m, n]) +
             ϵ33 Kδ_{y,n} (-δaa[b_m f g, k, l, i, j] + δaa[b_i f g, k, l, m, j]) +
             ϵ34 Kδ_{y,l,n} (δa[b_x b_m f g, k, j] - δa[b_k b_m f g, x, j])
          ),
        p[_δβ | _c, _a] → 0,
        p[δa[f_, i_, j_], a[g_, k_, l_]] :↦
         ϵ35 Kδ_{x,i} Kδ_{y,l} (-δa[b_k f g, i, j] + δa[b_i f g, k, j]),
        p[ca[f_, m_, i_, j_], a[g_, k_, l_]] :↦ ϵ36 Kδ_{x,i} Kδ_{y,l}
          (-ca[b_k f g, m, i, j] + ca[b_i f g, m, k, j]),
        p[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] :↦
         ϵ37 Kδ_{x,i} Kδ_{y,l} (-δaa[b_k f g, i, j, m, n] + δaa[b_i f g, k, j, m, n]) +
          ϵ38 Kδ_{x,m} Kδ_{y,l} (-δaa[b_k f g, i, j, m, n] + δaa[b_m f g, i, j, k, n])
       }]]]];
htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];
```

$t_1\, h_1\, t_2\, h_2 \to t_1\, t_2\, h_1\, h_2 \to t_2\, t_1\, h_1\, h_2 \to t_2\, t_1\, h_2\, h_1 \to t_2\, h_2\, t_1\, h_1$:

```
db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
    htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] // hm[x, h, x],
    tb[x][u, v // hσ[x, h]] // hm[x, h, x],
    hb[x][u, v // tσ[x, t]] // tm[t, x, x],
    thb[x, x][u // hσ[x, h], v // tσ[x, t]] // tm[t, x, x] // hm[x, h, x]
   ]];
```

```
bb[S_List] := Module[{w, bar, t, n = 0},
   bar[x_] := -x;
   w = #2 // dσ[S, bar /@ S];
   Sum[
    t = db[S[[k]]][#1, w // dσ[bar[S[[k]]], S[[k]]]];
    Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]]], {i, 1, k - 1}];
    Do[t = t // dm[S[[i]], bar[S[[i]]], S[[i]]], {i, k + 1, Length@S}];
    t,
    {k, Length@S}
   ]
  ] &
bb[S___] := bb[{S}]
```

ct (contract)

```
ct::usage =
  "ct[L,R,{h,t}] contracts the head h in L with the tail t in R. ct[L,R] assumes
    that both h and t are 0. When ambiguous, L is placeed below R.";
ct[L_, R_] := ct[L, R, {0, 0}];
ct[UU[L_], UU[R_], {h_, t_}] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
       p[_β | _δβ, _] → 0,
       p[a[f_, i_, h], β[g_]] :> β[f b_i ((∂_{b_t} g) /. b_t → 0)],
       p[a[f_, i_, h], a[g_, t, j_]] :> a[f (g /. b_t → 0), i, j],
       p[a[f_, i_, h], a[g_, j_, k_]] :> a[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
       p[a[f_, i_, h], c[g_, j_]] :> c[f b_i ((∂_{b_t} g) /. b_t → 0), j],
       p[a[f_, i_, h], δa[g_, t, j_]] :> δa[f (g /. b_t → 0), i, j],
       p[a[f_, i_, h], δa[g_, j_, k_]] :> δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
       p[a[f_, i_, h], ca[g_, k_, t, j_]] :> ca[f (g /. b_t → 0), k, i, j],
       p[a[f_, i_, h], ca[g_, l_, j_, k_]] :> ca[f b_i ((∂_{b_t} g) /. b_t → 0), l, j, k],
       p[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
       p[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] :> δaa[f (g /. b_t → 0), i, j, k, l],
       p[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] :> δaa[f (g /. b_t → 0), j, k, i, l],
       p[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] :>
        δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
       p[a[__], _] → 0,
       p[c[f_, h], β[g_]] :> δβ[f ((∂_{b_t} g) /. b_t → 0)],
       p[_c, _β] → 0,
       p[c[f_, h], a[g_, t, j_]] :> c[f (g /. b_t → 0), j],
       p[c[f_, h], a[g_, j_, k_]] :> δa[f ((∂_{b_t} g) /. b_t → 0), j, k],
       p[_c, _a] → 0,
       p[_c | _δa | _ca | _δaa, _δβ | _c | _δa | _ca | _δaa] → 0,
       p[δa[f_, i_, h], β[g_]] :> δβ[f b_i ((∂_{b_t} g) /. b_t → 0)],
```

```
 p[δa[f_, i_, h], a[g_, t, j_]] :> δa[f (g /. b_t → 0), i, j],
 p[δa[f_, i_, h], a[g_, j_, k_]] :> δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
 p[_δa, _] → 0,
 p[ca[_, h, _, h], _] → 0,
 p[ca[f_, h, i_, j_], β[g_]] :> δa[f ((∂_{b_t} g) /. b_t → 0), i, j],
 p[ca[f_, i_, j_, h], β[g_]] :> c[f b_j ((∂_{b_t} g) /. b_t → 0), i],
 p[ca[f_, h, i_, j_], a[g_, t, k_]] :> ca[f (g /. b_t → 0), k, i, j],
 p[ca[f_, h, i_, j_], a[g_, k_, l_]] :> δaa[f ((∂_{b_t} g) /. b_t → 0), i, j, k, l],
 p[ca[f_, i_, j_, h], a[g_, t, k_]] :> ca[f (g /. b_t → 0), i, j, k],
 p[ca[f_, i_, j_, h], a[g_, k_, l_]] :> ca[f b_j ((∂_{b_t} g) /. b_t → 0), i, k, l],
 p[_ca, _] → 0,
 p[δaa[_, _, h, _, h], _] → 0,
 p[δaa[f_, i_, h, j_, k_], β[g_]] :> δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
 p[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] :> δaa[f (g /. b_t → 0), i, l, j, k],
 p[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] :>
  δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
 p[δaa[f_, i_, j_, k_, h], β[g_]] :> δa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j],
 p[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] :> δaa[f (g /. b_t → 0), i, j, k, l],
 p[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] :>
  δaa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j, l, m],
 p[_δaa, _] → 0
}]]];
```