

Pensieve header: Scatter and Glow in OneCo. Continues pensieve://2016-01/, continued pensieve://2016-03/.

In the U(T)U(H) conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-02"]
```

I ought to be able to replace $c[i]$ with $a[c,i]$, and ca with aa , everywhere!

Generalities

```
DQ[is___] := (Sort[{is}] === Union[{is}]);
OQ[is___] := OrderedQ[{is}];
(* tests for non-strict ordering. Also true when {is} is {i,i}. *)
Kδis_ := KroneckerDelta[1, Length[Union[{is}]]];

Simp[expr_] := Expand[expr];
S[expr_] :=
  expr /. (λβ | λa | λδβ | λδa | λc | λca | λδaa) => MapAt[Simp, λ, 1];

AutoCollecting[λ_] := (
  λ /: λ[0, ___] = 0;
  λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f+g], r];
  λ /: g_ * λ[f_, r___] := λ[Simp[g f], r];
);
AutoCollecting /@ {β, a, δβ, c, δa, ca, δaa};
UU /: UU[x_] + UU[y_] := UU[x+y];
UU /: a_ * UU[x_] := UU[Expand[a x]];

Υ[f_, j_, k_] := δa[f, j, k] - c[ε48 bj f, k];
Υa[f_, j_, k_, l_, m_] := δaa[f, j, k, l, m] - ca[ε48 bj f, k, l, m];
```

Bases

```

UUBasis[T_List, H_List, f_] := Module[
  {ff, n = 0, h, t, h1, h2},
  ff := f_{++n}@@Table[b_t, {t, T}];
  UU /@ Flatten@{
     $\beta$ [ff],
    Table[{a[ff, t, h],  $\delta a$ [ff, t, h]}, {t, T}, {h, H}],
     $\delta\beta$ [ff],
    Table[c[ff, h], {h, H}],
    Table[ca[ff, h1, t, h2], {h1, H}, {t, T}, {h2, H}],
    Table[ $\delta aa$ [ff, T[[i]], H[[j]], T[[k]], H[[l]],
      {k, Length@T}, {i, k}, {l, Length@H}, {j, l}]
    ] /. 1_[_]_ → 1
  ];
UUBasis[S_List, f_] := UUBasis[S, S, f];
UUBasis[n_Integer, m_Integer, f_] := UUBasis[Range@n, Range@m, f];
UUBasis[n_Integer, f_] := UUBasis[Range@n, f];

```

δaa relations

Switch from thth to tthh indexing? (not for the moment)

```

UU[expr_] // S := UU[S[expr]]; (* Temporary fixture! *)
UU[expr_] // S := UU[S[expr] /. {
   $\delta aa$ [f_, i_, j_, k_, l_] /; !OQ[j, l]  $\Rightarrow$   $\delta aa$ [f, k, l, i, j],
   $\delta aa$ [f_, i_, j_, k_, l_] /; !OQ[i, k]  $\wedge$  DQ[j, l]  $\wedge$  OQ[j, l]  $\Rightarrow$ 
     $\delta aa$ [f, i, l, k, j] + ca[ $\epsilon_4 b_k f, l, i, j$ ] + ca[- $\epsilon_4 b_i f, l, k, j$ ] +
    ca[- $\epsilon_4 b_k f, j, i, l$ ] + ca[ $\epsilon_4 b_i f, j, k, l$ ],
   $\delta aa$ [f_, i_, k_, j_, k_] /; !OQ[i, j]  $\Rightarrow$   $\delta aa$ [f, j, k, i, k] +
     $\delta a$ [- $\epsilon_2 b_i f, j, k$ ] +  $\delta a$ [ $\epsilon_2 b_j f, i, k$ ]
}]]];

```

tm, hm, hts, dm

```

UU[expr_] // tm[x_, y_, z_] := S[UU[Expand[expr /. {
  a[f_, x, j_] := a[f, z, j] + ε13 γ[∂by f, z, j],
  a[f_, y, j_] := a[f, z, j],
  δa[f_, x | y, j_] := δa[f, z, j],
  ca[f_, i_, x | y, j_] := ca[f, i, z, j],
  δaa[f_, i_, j_, k_, l_] :=
    δaa[f, i // Replace[x | y → z], j, k // Replace[x | y → z], l]
} /. bx|y → bz]]];

UU[expr_] // hm[x_, y_, z_] := S[UU[Expand[expr /. {
  a[f_, i_, x | y] := a[f, i, z],
  c[f_, x | y] := c[f, z],
  δa[f_, i_, x | y] := δa[f, i, z],
  ca[f_, y, j_, x] := ca[f, z, j, z] + ε12 γ[f, j, z],
  ca[f_, i_, j_, k_] :=
    ca[f, i // Replace[x | y → z], j, k // Replace[x | y → z]],
  δaa[f_, i_, y, k_, x] := δaa[f, k, z, i, z],
  δaa[f_, i_, j_, k_, l_] :=
    δaa[f, i, j // Replace[x | y → z], k, l // Replace[x | y → z]]
}]]];

UU[expr_] // hts[y_, x_] := S[UU[Expand[expr /. {
  a[f_, i_, j_] := a[f, i, j] - ε11 Kδj,y γ[∂bx f, i, y] -
    Kδi,x Kδj,y (ε5 β[f bx] + ε6 c[f, y] - ε7 δβ[bx ∂bx f]),
  δa[f_, x, y] := δa[f, x, y] - ε8 δβ[f bx],
  ca[f_, i_, j_, k_] :=
    ca[f, i, j, k] + ε10 Kδi,y Kδj,x γ[f, x, k] + Kδj,x Kδk,y c[-ε9 f bx, i],
  δaa[f_, i_, j_, k_, l_] := δaa[f, i, j, k, l] + ε14 Kδi,x Kδj,y δa[-bx f, k, l] +
    ε3 Kδi,x Kδl,y (-δa[bx f, x, j] + δa[bx f, k, j]) +
    ε1 Kδk,x Kδj,y (δa[bi f, x, l] - δa[bx f, i, l]) +
    ε15 Kδk,x Kδl,y δa[-bx f, i, j] + ε16 Kδi,x Kδj,l,y δβ[bi bk f]
}]]];

dm[x_, y_, z_][expr_] := expr // hts[x, y] // tm[x, y, z] // hm[x, y, z]

```

$t\sigma, h\sigma, d\sigma$ on $\{\beta, a, \delta\beta, c, \delta a, ca, \delta aa\}$

```

tσ[x_List, y_List][expr_] := Module[{rule = Thread[x → y]},
  expr /. b_i :=> b_i /. rule /. {
    a[f_, i_, j_] :=> a[f, i /. rule, j],
    δa[f_, i_, j_] :=> δa[f, i /. rule, j],
    ca[f_, i_, j_, k_] :=> ca[f, i, j /. rule, k],
    δaa[f_, i_, j_, k_, l_] :=> δaa[f, i /. rule, j, k /. rule, l]
  }
];

tσ[x_, y_][expr_] := tσ[{x}, {y}][expr];

hσ[x_List, y_List][expr_] := Module[{rule = Thread[x → y]},
  S[expr /. {
    a[f_, i_, j_] :=> a[f, i, j /. rule],
    c[f_, i_] :=> c[f, i /. rule],
    δa[f_, i_, j_] :=> δa[f, i, j /. rule],
    ca[f_, i_, j_, k_] :=> ca[f, i /. rule, j, k /. rule],
    δaa[f_, i_, j_, k_, l_] :=> δaa[f, i, j /. rule, k, l /. rule]
  }]
];

hσ[x_, y_][expr_] := hσ[{x}, {y}][expr];
dσ[x_, y_][expr_] := expr // tσ[x, y] // hσ[x, y];

```

tb, hb, thb, htb, db, bb on $\{\beta, a, \delta\beta, c, \delta a, ca, \delta aa\}$

```

tb[x_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
  p[0, _] → 0, p[_ , 0] → 0,
  p[_β | _δβ | _c | _δa | _ca | _δaa, _β | _δβ | _c | _δa | _ca | _δaa] → 0,
  p[u_β | u_δβ | u_c | u_δa | u_ca | u_δaa, v_a] :=> -p[v, u]
} /. {
  p[a[f_, x, j_], u_] :=> (u /. {
    β[g_] :=> ε39 γ[f ∂bx g, x, j],
    a[g_, k_, l_] :=> ε40 γa[f ∂bx g, x, j, k, l] + Kδx,k (-γa[ε41 g ∂bx f, k,
      l, x, j] + ca[ε17 f g, l, x, j] - ca[ε17 f g, j, k, l]),
    _ → 0
  })],
  p[a[f_, j_, k_], a[g_, x, l_]] :=> -γa[ε42 g ∂bx f, x, l, j, k],
  p[_ , _] → 0
}]]];

```

```

hb[y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
  p[0, _] → 0, p[_ , 0] → 0,
  p[_β | _δβ, _] → 0,
  p[_ , _β | _δβ] → 0,
  p[_c | _δa | _ca | _δaa, _c | _δa | _ca | _δaa] → 0,
  p[u_c | u_δa | u_ca | u_δaa, v_a] := -p[v, u]
} /. {
p[a[f_, i_, y], u_] := (u /. {
  a[g_, j_, k_] := ε18 Kδy,k (a[bj f g, i, y] - a[bi f g, j, k]),
  c[g_, j_] := ε19 Kδy,j γ[f g, i, j],
  δa[g_, j_, k_] := ε20 Kδy,k (δa[bj f g, i, y] - δa[bi f g, j, k]),
  ca[g_, j_, k_, l_] := Kδy,j γa[ε21 f g, i, j, k, l] +
    Kδy,l (ca[ε22 bk f g, j, i, y] - ca[ε22 bi f g, j, k, l]),
  δaa[g_, j_, k_, l_, m_] := ε23 Kδy,k (δaa[bj f g, i, y, l, m] - δaa[bi f g, j,
    k, l, m]) + ε24 Kδy,m (δaa[bl f g, j, k, i, y] - δaa[bi f g, j, k, l, m])
}),
_p → 0
}]]];

```

```

thb[x_, y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Expand[Distribute[p[L, R]] /. {
  p[0, _] → 0, p[_ , 0] → 0,
  p[_β | _δβ | _c | _δa | _ca | _δaa, _β | _δβ | _c | _δa | _ca | _δaa] → 0,
  p[_a, _β | _δβ] → 0,
  p[β[f_], a[g_, i_, j_]] := Kδy,j γ[ε43 g ∂bx f, i, y],
  p[a[f_, i_, j_], a[g_, k_, l_]] := Kδy,l (
    γa[ε44 g ∂bx f, k, l, i, j] + Kδx,i (
      γ[-ε45 bk g ∂bx f, i, j] + δa[ε46 bk g ∂bx f,
        i, j] - δa[ε47 bi g ∂bx f, k, j] - a[ε25 bk f g, i, j] + a[
          ε25 bi f g, k, j] + ca[ε26 f g, j, k, l] - ca[ε26 f g, l, k, j]
        )
    ),
  p[a[f_, i_, j_], c[g_, k_]] := -ε27 Kδi,x Kδk,y γ[f g, i, j],
  p[a[f_, i_, j_], δa[g_, k_, l_]] :=
    ε28 Kδx,i Kδy,l (-δa[bk f g, i, j] + δa[bi f g, k, j]),
  p[a[f_, i_, j_], ca[g_, k_, l_, m_]] := Kδx,i (
    -ε29 Kδy,k γa[f g, i, j, l, m] + ε30 Kδy,m
      (-ca[bl f g, k, i, j] + ca[bi f g, k, l, j]) - ε31 Kδy,k,m γ[bl f g, x, j]
    ),
  p[a[f_, i_, j_], δaa[g_, k_, l_, m_, n_]] := Kδx,i (
    ε32 Kδy,l (-δaa[bk f g, i, j, m, n] + δaa[bi f g, k, j, m, n]) +
    ε33 Kδy,n (-δaa[bm f g, k, l, i, j] + δaa[bi f g, k, l, m, j]) +
    ε34 Kδy,l,n (δa[bx bm f g, k, j] - δa[bk bm f g, x, j])
  ),
  p[_δβ | _c, _a] → 0,
  p[δa[f_, i_, j_], a[g_, k_, l_]] :=
    ε35 Kδx,i Kδy,l (-δa[bk f g, i, j] + δa[bi f g, k, j]),
  p[ca[f_, m_, i_, j_], a[g_, k_, l_]] := ε36 Kδx,i Kδy,l
    (-ca[bk f g, m, i, j] + ca[bi f g, m, k, j]),
  p[δaa[f_, i_, j_, m_, n_], a[g_, k_, l_]] :=
    ε37 Kδx,i Kδy,l (-δaa[bk f g, i, j, m, n] + δaa[bi f g, k, j, m, n]) +
    ε38 Kδx,m Kδy,l (-δaa[bk f g, i, j, m, n] + δaa[bm f g, i, j, k, n])
}]]];

```

```

htb[x_, y_][L_UU, R_UU] := -thb[y, x][R, L];

```

$$t_1 h_1 t_2 h_2 \rightarrow t_1 t_2 h_1 h_2 \rightarrow t_2 t_1 h_1 h_2 \rightarrow t_2 t_1 h_2 h_1 \rightarrow t_2 h_2 t_1 h_1 :$$

```

db[x_][u_UU, v_UU] := Module[{t, h}, Plus[
  htb[x, x][u // tσ[x, t], v // hσ[x, h]] // tm[t, x, x] // hm[x, h, x],
  tb[x][u, v // hσ[x, h]] // hm[x, h, x],
  hb[x][u, v // tσ[x, t]] // tm[t, x, x],
  thb[x, x][u // hσ[x, h], v // tσ[x, t]] // tm[t, x, x] // hm[x, h, x]
]];

```

```

bb[S_List] := Module[{w, bar, t, n = 0},
  bar[x_] := -x;
  w = #2 // do[S, bar /@ S];
  Sum[
    t = db[S[[k]]][#1, w // do[bar[S[[k]], S[[k]]];
    Do[t = t // dm[bar[S[[i]], S[[i]], S[[i]], {i, 1, k - 1}];
    Do[t = t // dm[S[[i]], bar[S[[i]], S[[i]], {i, k + 1, Length@S}];
    t,
    {k, Length@S}
  ]
] &
bb[S___] := bb[{S}]

```

ct (contract)

ct::usage =

"ct[L,R,{h,t}] contracts the head h in L with the tail t in R. ct[L,R] assumes that both h and t are 0. When ambiguous, L is placed below R.";

```
ct[L_, R_] := ct[L, R, {0, 0}];
```

```

ct[UU[L_], UU[R_], {h_, t_}] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
  p[_β | _δβ, _] → 0,
  p[a[f_, i_, h], β[g_]] ⇒ β[f b_i ((∂btg) /. bt → 0)],
  p[a[f_, i_, h], a[g_, t, j_]] ⇒ a[f (g /. bt → 0), i, j],
  p[a[f_, i_, h], a[g_, j_, k_]] ⇒ a[f b_i ((∂btg) /. bt → 0), j, k],
  p[a[f_, i_, h], c[g_, j_]] ⇒ c[f b_i ((∂btg) /. bt → 0), j],
  p[a[f_, i_, h], δa[g_, t, j_]] ⇒ δa[f (g /. bt → 0), i, j],
  p[a[f_, i_, h], δa[g_, j_, k_]] ⇒ δa[f b_i ((∂btg) /. bt → 0), j, k],
  p[a[f_, i_, h], ca[g_, k_, t, j_]] ⇒ ca[f (g /. bt → 0), k, i, j],
  p[a[f_, i_, h], ca[g_, l_, j_, k_]] ⇒ ca[f b_i ((∂btg) /. bt → 0), l, j, k],
  p[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
  p[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] ⇒ δaa[f (g /. bt → 0), i, j, k, l],
  p[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] ⇒ δaa[f (g /. bt → 0), j, k, i, l],
  p[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] ⇒
    δaa[f b_i ((∂btg) /. bt → 0), j, k, l, m],
  p[a[_], _] → 0,
  p[c[f_, h], β[g_]] ⇒ δβ[f ((∂btg) /. bt → 0)],
  p[_c, _β] → 0,
  p[c[f_, h], a[g_, t, j_]] ⇒ c[f (g /. bt → 0), j],
  p[c[f_, h], a[g_, j_, k_]] ⇒ δa[f ((∂btg) /. bt → 0), j, k],
  p[_c, _a] → 0,
  p[_c | _δa | _ca | _δaa, _δβ | _c | _δa | _ca | _δaa] → 0,
  p[δa[f_, i_, h], β[g_]] ⇒ δβ[f b_i ((∂btg) /. bt → 0)],

```

```

p[δa[f_, i_, h], a[g_, t, j_]] := δa[f (g /. b_t → 0), i, j],
p[δa[f_, i_, h], a[g_, j_, k_]] := δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
p[_δa, _] → 0,
p[ca[_ , h, _ , h], _] → 0,
p[ca[f_, h, i_, j_], β[g_]] := δa[f ((∂_{b_t} g) /. b_t → 0), i, j],
p[ca[f_, i_, j_, h], β[g_]] := c[f b_j ((∂_{b_t} g) /. b_t → 0), i],
p[ca[f_, h, i_, j_], a[g_, t, k_]] := ca[f (g /. b_t → 0), k, i, j],
p[ca[f_, h, i_, j_], a[g_, k_, l_]] := δaa[f ((∂_{b_t} g) /. b_t → 0), i, j, k, l],
p[ca[f_, i_, j_, h], a[g_, t, k_]] := ca[f (g /. b_t → 0), i, j, k],
p[ca[f_, i_, j_, h], a[g_, k_, l_]] := ca[f b_j ((∂_{b_t} g) /. b_t → 0), i, k, l],
p[_ca, _] → 0,
p[δaa[_ , _ , h, _ , h], _] → 0,
p[δaa[f_, i_, h, j_, k_], β[g_]] := δa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k],
p[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] := δaa[f (g /. b_t → 0), i, l, j, k],
p[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] :=
  δaa[f b_i ((∂_{b_t} g) /. b_t → 0), j, k, l, m],
p[δaa[f_, i_, j_, k_, h], β[g_]] := δa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j],
p[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] := δaa[f (g /. b_t → 0), i, j, k, l],
p[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] :=
  δaa[f b_k ((∂_{b_t} g) /. b_t → 0), i, j, l, m],
p[_δaa, _] → 0
}}]];

```