

Pensieve header: Testing notebook for Scatter and Glow in OneCo.

In the U(T)U(H) conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-02"];
<< SnG.m
```

Bases

```
UUBasis[{1, 2}, {3, 4}, f]
```

```
{UU[β[f1[b1, b2]]], UU[a[f2[b1, b2], 1, 3]], UU[δa[f3[b1, b2], 1, 3]],
UU[a[f4[b1, b2], 1, 4]], UU[δa[f5[b1, b2], 1, 4]], UU[a[f6[b1, b2], 2, 3]],
UU[δa[f7[b1, b2], 2, 3]], UU[a[f8[b1, b2], 2, 4]], UU[δa[f9[b1, b2], 2, 4]],
UU[δβ[f10[b1, b2]]], UU[c[f11[b1, b2], 3]], UU[c[f12[b1, b2], 4]],
UU[ca[f13[b1, b2], 3, 1, 3]], UU[ca[f14[b1, b2], 3, 1, 4]],
UU[ca[f15[b1, b2], 3, 2, 3]], UU[ca[f16[b1, b2], 3, 2, 4]],
UU[ca[f17[b1, b2], 4, 1, 3]], UU[ca[f18[b1, b2], 4, 1, 4]], UU[ca[f19[b1, b2], 4, 2, 3]],
UU[ca[f20[b1, b2], 4, 2, 4]], UU[δaa[f21[b1, b2], 1, 3, 1, 3]],
UU[δaa[f22[b1, b2], 1, 3, 1, 4]], UU[δaa[f23[b1, b2], 1, 4, 1, 4]],
UU[δaa[f24[b1, b2], 1, 3, 2, 3]], UU[δaa[f25[b1, b2], 1, 3, 2, 4]],
UU[δaa[f26[b1, b2], 1, 4, 2, 4]], UU[δaa[f27[b1, b2], 2, 3, 2, 3]],
UU[δaa[f28[b1, b2], 2, 3, 2, 4]], UU[δaa[f29[b1, b2], 2, 4, 2, 4]]}
```

Meta-associativity for tm

```
((# // tm[s1, s2, s1] // tm[s1, s3, s1]) ==
  (# // tm[s2, s3, s2] // tm[s1, s2, s1])) & /@
  UUBasis[{s1, s2, s3, s4}, {s5, s6}, f] // Union
{True}
```

Meta-associativity for hm

```
((# // hm[s1, s2, s1] // hm[s1, s3, s1]) ==
  (# // hm[s2, s3, s2] // hm[s1, s2, s1])) & /@
  UUBasis[{s5, s6}, {s1, s2, s3, s4}, f] // Union
{True}
```

Compatibility between tm and hts

```
(( (# // tm[s1, s2, s1] // hts[s4, s1]) ==
  (# // hts[s4, s1] // hts[s4, s2] // tm[s1, s2, s1])) & /@
  UUBasis[{s1, s2, s3}, {s4, s5}, f] // Union
{True}
```

Compatibility between hm and hts

```
(( (# // hm[s1, s2, s1] // hts[s1, s4]) ==
  (# // hts[s2, s4] // hts[s1, s4] // hm[s1, s2, s1])) & /@
  UUBasis[{s4, s5}, {s1, s2, s3}, f] // Union
{True}
```

Meta-associativity for dm

```
(( (# // dm[s1, s2, s1] // dm[s1, s3, s1]) ==
  (# // dm[s2, s3, s2] // dm[s1, s2, s1])) & /@
  UUBasis[{s1, s2, s3, s4}, f] // Union
{True}
```

Meta-AS and meta-Jacobi for tb

```
tbAS[u_, v_] := tb[s0][u, v] + tb[s0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → tbAS[##] &,
    UUBasis[{s0, s1}, {s1, s2}, f], UUBasis[{s0, s2}, {s3, s4}, f]
  ]],
  _ → UU[0]
]
{}
```

```

tbJacobi[u_, v_, w_] :=
  tb[s0][u, tb[s0][v, w]] + tb[s0][v, tb[s0][w, u]] + tb[s0][w, tb[s0][u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → tbJacobi[##] &,
    UUBasis[{s0, s1}, {s1, s2}, f],
    UUBasis[{s0, s2}, {s3, s4}, g], UUBasis[{s0, s3}, {s5, s6}, h]
  ]],
  _ → UU[0]
]
{}

```

Compatibility of tb and tm

```

tbtm[u_, v_] := Plus[
  tb[s1][u, tσ[s2, -s2][v]] // tm[s2, -s2, s2],
  -tb[s1][u, tσ[s2, -s2][v]] // tm[-s2, s2, s2],
  -tb[s2][u, tσ[s1, -s1][v]] // tm[s1, -s1, s1],
  tb[s2][u, tσ[s1, -s1][v]] // tm[-s1, s1, s1]
];
DeleteCases[
  Flatten[Outer[
    {##} → tbtm[##] &,
    UUBasis[{s1, s2, s3}, {s1, s2}, f], UUBasis[{s1, s2, s4}, {s3, s4}, f]
  ]],
  _ → UU[0]
]
{}

```

Meta-AS and meta-Jacobi for hb

```

hbAS[u_, v_] := hb[s0][u, v] + hb[s0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → hbAS[##] &,
    UUBasis[{s1, s2}, {0, 1}, f], UUBasis[{s3, s4}, {s0, s2}, g]
  ]],
  _ → UU[0]
]
{}

```

```

hbJacobi[u_, v_, w_] :=
  hb[s0][u, hb[s0][v, w]] + hb[s0][v, hb[s0][w, u]] + hb[s0][w, hb[s0][u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → hbJacobi[##] &,
    UUBasis[{s1, s2}, {s0, s1}, f],
    UUBasis[{s3, s4}, {s0, s2}, g], UUBasis[{s5, s6}, {s0, s3}, h]
  ]],
  _ → UU[0]
]
{}

```

Compatibility of tb and tm

```

hbhm[u_, v_] := Plus[
  hb[s1][u, hσ[s2, -s2][v]] // hm[s2, -s2, s2],
  -hb[s1][u, hσ[s2, -s2][v]] // hm[-s2, s2, s2],
  -hb[s2][u, hσ[s1, -s1][v]] // hm[s1, -s1, s1],
  hb[s2][u, hσ[s1, -s1][v]] // hm[-s1, s1, s1]
];
DeleteCases[
  Flatten[Outer[
    {##} → hbhm[##] &,
    UUBasis[{s1, s2}, {s1, s2, s3}, f], UUBasis[{s3, s4}, {s1, s2, s4}, g]
  ]],
  _ → UU[0]
]
{}

```

Meta-Jacobi for thb/tb and for thb/hb

```

tthJacobi[u_, v_, w_] := Plus[
  -thb[s0, s0][tb[s0][u, v], w] +
  tb[s0][thb[s0, s0][u, w], v] - thb[s0, s0][v, thb[s0, s0][u, w]] +
  tb[s0][u, thb[s0, s0][v, w]] + thb[s0, s0][u, thb[s0, s0][v, w]]
];
DeleteCases[
  Flatten[Outer[
    {##} → tthJacobi[##] &,
    UUBasis[{s0, s1}, {s1, s2}, f],
    UUBasis[{s0, s2}, {s3, s4}, g], UUBasis[{s3, s4}, {s0, s5}, h]
  ]],
  _ → UU[0]
]
{}

thhJacobi[u_, v_, w_] := Plus[
  -thb[s0, s0][u, hb[s0][v, w]] +
  hb[s0][thb[s0, s0][u, v], w] + thb[s0, s0][thb[s0, s0][u, v], w] +
  hb[s0][v, thb[s0, s0][u, w]] - thb[s0, s0][thb[s0, s0][u, w], v]
];
DeleteCases[
  Flatten[Outer[
    {##} → thhJacobi[##] &,
    UUBasis[{s0, s1}, {s1, s2}, f],
    UUBasis[{s2, s3}, {s0, s3}, g], UUBasis[{s4, s5}, {s0, s4}, h]
  ]],
  _ → UU[0]
]
{}

```

Meta-AS and meta-Jacobi for db

```

dbAS[u_, v_] := db[s0][u, v] + db[s0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → dbAS[##] &,
    UUBasis[{s0, s1, s2}, f], UUBasis[{s0, s3, s4}, g]
  ]],
  _ → UU[0]
]
{}

dbJacobi[u_, v_, w_] :=
  db[s0][u, db[s0][v, w]] + db[s0][v, db[s0][w, u]] + db[s0][w, db[s0][u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → dbJacobi[##] &,
    UUBasis[{s0, s1, s2}, f], UUBasis[{s0, s3, s4}, g], UUBasis[{s0, s5, s6}, h]
  ]],
  _ → UU[0]
]
{}

```

AS and Jacobi for bb

```

bb4 = bb[s1, s2, s3, s4];
bbAS[u_, v_] := bb4[u, v] + bb4[v, u];
DeleteCases[
  Flatten[Outer[
    {##} → bbAS[##] &,
    UUBasis[{s1, s2}, f], UUBasis[{s1, s2}, g]
  ]],
  _ → UU[0]
]
{
  {UU[a[f2[b_s1, b_s2], s1, s1]], UU[a[g6[b_s1, b_s2], s2, s1]]} →
  UU[δa[-2 b_s1 g6[b_s1, b_s2] f2^(0,1)[b_s1, b_s2], s2, s1] +
  δa[2 b_s2 g6[b_s1, b_s2] f2^(0,1)[b_s1, b_s2], s1, s1]],
  {UU[a[f4[b_s1, b_s2], s1, s2]], UU[a[g6[b_s1, b_s2], s2, s1]]} →
  UU[c[-2 b_s1 f4[b_s1, b_s2] g6[b_s1, b_s2], s2] + c[2 b_s2 f4[b_s1, b_s2] g6[b_s1, b_s2], s1] +
  δa[-2 b_s1 g6[b_s1, b_s2] f4^(0,1)[b_s1, b_s2] - 2 b_s1 f4[b_s1, b_s2] g6^(0,1)[b_s1, b_s2], s2, s2] +
  δa[2 b_s2 g6[b_s1, b_s2] f4^(0,1)[b_s1, b_s2] + 2 b_s2 f4[b_s1, b_s2] g6^(0,1)[b_s1, b_s2], s1, s2]],
  {UU[a[f4[b_s1, b_s2], s1, s2]], UU[a[g8[b_s1, b_s2], s2, s2]]} →

```

$$\begin{aligned}
& \text{UU}[\delta a[-b_{s1} f_4[b_{s1}, b_{s2}] g_8^{(1,0)}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \epsilon_1 (\delta a[b_{s1} f_4[b_{s1}, b_{s2}] g_8^{(1,0)}[b_{s1}, b_{s2}], s2, s2] + \delta a[-b_{s2} f_4[b_{s1}, b_{s2}] g_8^{(1,0)}[b_{s1}, b_{s2}], \\
& \quad \quad s1, s2]) + \delta a[b_{s2} f_4[b_{s1}, b_{s2}] g_8^{(1,0)}[b_{s1}, b_{s2}], s1, s2]), \\
& \{ \text{UU}[a[f_4[b_{s1}, b_{s2}], s1, s2]], \text{UU}[ca[g_{16}[b_{s1}, b_{s2}], s1, s2, s2]] \} \rightarrow \\
& \text{UU}[\delta a[b_{s1} f_4[b_{s1}, b_{s2}] g_{16}[b_{s1}, b_{s2}], s2, s2] + \delta a[-b_{s2} f_4[b_{s1}, b_{s2}] g_{16}[b_{s1}, b_{s2}], s1, s2] + \\
& \quad \epsilon_1 (\delta a[-b_{s1} f_4[b_{s1}, b_{s2}] g_{16}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \quad \delta a[b_{s2} f_4[b_{s1}, b_{s2}] g_{16}[b_{s1}, b_{s2}], s1, s2])], \\
& \{ \text{UU}[a[f_4[b_{s1}, b_{s2}], s1, s2]], \text{UU}[ca[g_{19}[b_{s1}, b_{s2}], s2, s2, s1]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s1} f_4[b_{s1}, b_{s2}] g_{19}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \delta a[-2 b_{s2} f_4[b_{s1}, b_{s2}] g_{19}[b_{s1}, b_{s2}], s1, s2]), \\
& \{ \text{UU}[a[f_4[b_{s1}, b_{s2}], s1, s2]], \text{UU}[\delta aa[g_{24}[b_{s1}, b_{s2}], s1, s1, s2, s1]] \} \rightarrow \\
& \text{UU}[\delta \beta[-2 b_{s1}^2 b_{s2} f_4[b_{s1}, b_{s2}] g_{24}[b_{s1}, b_{s2}]]], \\
& \{ \text{UU}[a[f_4[b_{s1}, b_{s2}], s1, s2]], \text{UU}[\delta aa[g_{27}[b_{s1}, b_{s2}], s2, s1, s2, s1]] \} \rightarrow \\
& \text{UU}[\delta a[-4 b_{s1} b_{s2} f_4[b_{s1}, b_{s2}] g_{27}[b_{s1}, b_{s2}], s2, s1] + \\
& \quad \delta a[4 b_{s2}^2 f_4[b_{s1}, b_{s2}] g_{27}[b_{s1}, b_{s2}], s1, s1] + \delta \beta[-4 b_{s1} b_{s2}^2 f_4[b_{s1}, b_{s2}] g_{27}[b_{s1}, b_{s2}]]], \\
& \{ \text{UU}[a[f_4[b_{s1}, b_{s2}], s1, s2]], \text{UU}[\delta aa[g_{28}[b_{s1}, b_{s2}], s2, s1, s2, s2]] \} \rightarrow \\
& \text{UU}[\delta a[-b_{s1} b_{s2} f_4[b_{s1}, b_{s2}] g_{28}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \delta a[b_{s2}^2 f_4[b_{s1}, b_{s2}] g_{28}[b_{s1}, b_{s2}], s1, s2] + \epsilon_1 (\delta a[-b_{s1} b_{s2} f_4[b_{s1}, b_{s2}] g_{28}[b_{s1}, b_{s2}], \\
& \quad \quad s2, s2] + \delta a[b_{s2}^2 f_4[b_{s1}, b_{s2}] g_{28}[b_{s1}, b_{s2}], s1, s2])], \\
& \{ \text{UU}[a[f_6[b_{s1}, b_{s2}], s2, s1]], \text{UU}[a[g_2[b_{s1}, b_{s2}], s1, s1]] \} \rightarrow \\
& \text{UU}[\delta a[-2 b_{s1} f_6[b_{s1}, b_{s2}] g_2^{(0,1)}[b_{s1}, b_{s2}], s2, s1] + \\
& \quad \delta a[2 b_{s2} f_6[b_{s1}, b_{s2}] g_2^{(0,1)}[b_{s1}, b_{s2}], s1, s1]), \\
& \{ \text{UU}[a[f_6[b_{s1}, b_{s2}], s2, s1]], \text{UU}[a[g_4[b_{s1}, b_{s2}], s1, s2]] \} \rightarrow \\
& \text{UU}[c[-2 b_{s1} f_6[b_{s1}, b_{s2}] g_4[b_{s1}, b_{s2}], s2] + c[2 b_{s2} f_6[b_{s1}, b_{s2}] g_4[b_{s1}, b_{s2}], s1] + \\
& \quad \delta a[-2 b_{s1} g_4[b_{s1}, b_{s2}] f_6^{(0,1)}[b_{s1}, b_{s2}] - 2 b_{s1} f_6[b_{s1}, b_{s2}] g_4^{(0,1)}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \delta a[2 b_{s2} g_4[b_{s1}, b_{s2}] f_6^{(0,1)}[b_{s1}, b_{s2}] + 2 b_{s2} f_6[b_{s1}, b_{s2}] g_4^{(0,1)}[b_{s1}, b_{s2}], s1, s2]), \\
& \{ \text{UU}[a[f_6[b_{s1}, b_{s2}], s2, s1]], \text{UU}[ca[g_{17}[b_{s1}, b_{s2}], s2, s1, s1]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s1} f_6[b_{s1}, b_{s2}] g_{17}[b_{s1}, b_{s2}], s2, s1] + \\
& \quad \delta a[-2 b_{s2} f_6[b_{s1}, b_{s2}] g_{17}[b_{s1}, b_{s2}], s1, s1]), \\
& \{ \text{UU}[a[f_6[b_{s1}, b_{s2}], s2, s1]], \text{UU}[ca[g_{18}[b_{s1}, b_{s2}], s2, s1, s2]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s1} f_6[b_{s1}, b_{s2}] g_{18}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \delta a[-2 b_{s2} f_6[b_{s1}, b_{s2}] g_{18}[b_{s1}, b_{s2}], s1, s2]), \\
& \{ \text{UU}[a[f_6[b_{s1}, b_{s2}], s2, s1]], \text{UU}[\delta aa[g_{22}[b_{s1}, b_{s2}], s1, s1, s1, s2]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s1}^2 f_6[b_{s1}, b_{s2}] g_{22}[b_{s1}, b_{s2}], s2, s1] + \\
& \quad \delta a[-2 b_{s1} b_{s2} f_6[b_{s1}, b_{s2}] g_{22}[b_{s1}, b_{s2}], s1, s1]), \\
& \{ \text{UU}[a[f_6[b_{s1}, b_{s2}], s2, s1]], \text{UU}[\delta aa[g_{23}[b_{s1}, b_{s2}], s1, s2, s1, s2]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s1}^2 f_6[b_{s1}, b_{s2}] g_{23}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \delta a[-2 b_{s1} b_{s2} f_6[b_{s1}, b_{s2}] g_{23}[b_{s1}, b_{s2}], s1, s2] + \epsilon_1 (\delta a[2 b_{s1}^2 f_6[b_{s1}, b_{s2}] g_{23}[b_{s1}, b_{s2}], \\
& \quad \quad s2, s2] + \delta a[-2 b_{s1} b_{s2} f_6[b_{s1}, b_{s2}] g_{23}[b_{s1}, b_{s2}], s1, s2])], \\
& \{ \text{UU}[a[f_6[b_{s1}, b_{s2}], s2, s1]], \text{UU}[\delta aa[g_{26}[b_{s1}, b_{s2}], s1, s2, s2, s2]] \} \rightarrow \\
& \text{UU}[\delta a[-2 b_{s1} b_{s2} f_6[b_{s1}, b_{s2}] g_{26}[b_{s1}, b_{s2}], s2, s2] + \\
& \quad \delta a[2 b_{s2}^2 f_6[b_{s1}, b_{s2}] g_{26}[b_{s1}, b_{s2}], s1, s2]), \\
& \{ \text{UU}[a[f_8[b_{s1}, b_{s2}], s2, s2]], \text{UU}[a[g_4[b_{s1}, b_{s2}], s1, s2]] \} \rightarrow
\end{aligned}$$

$$\begin{aligned}
& \text{UU}[\delta a[-b_{s_1} g_4[b_{s_1}, b_{s_2}] f_8^{(1,0)}[b_{s_1}, b_{s_2}], s_2, s_2] + \\
& \quad \epsilon_1 (\delta a[b_{s_1} g_4[b_{s_1}, b_{s_2}] f_8^{(1,0)}[b_{s_1}, b_{s_2}], s_2, s_2] + \delta a[-b_{s_2} g_4[b_{s_1}, b_{s_2}] f_8^{(1,0)}[b_{s_1}, b_{s_2}], \\
& \quad \quad s_1, s_2]) + \delta a[b_{s_2} g_4[b_{s_1}, b_{s_2}] f_8^{(1,0)}[b_{s_1}, b_{s_2}], s_1, s_2]), \\
& \{ \text{UU}[ca[f_{16}[b_{s_1}, b_{s_2}], s_1, s_2, s_2]], \text{UU}[a[g_4[b_{s_1}, b_{s_2}], s_1, s_2]] \} \rightarrow \\
& \text{UU}[\delta a[b_{s_1} f_{16}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_2, s_2] + \delta a[-b_{s_2} f_{16}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_1, s_2] + \\
& \quad \epsilon_1 (\delta a[-b_{s_1} f_{16}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_2, s_2] + \\
& \quad \quad \delta a[b_{s_2} f_{16}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_1, s_2])], \\
& \{ \text{UU}[ca[f_{17}[b_{s_1}, b_{s_2}], s_2, s_1, s_1]], \text{UU}[a[g_6[b_{s_1}, b_{s_2}], s_2, s_1]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s_1} f_{17}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_2, s_1] + \\
& \quad \delta a[-2 b_{s_2} f_{17}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_1, s_1]), \\
& \{ \text{UU}[ca[f_{18}[b_{s_1}, b_{s_2}], s_2, s_1, s_2]], \text{UU}[a[g_6[b_{s_1}, b_{s_2}], s_2, s_1]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s_1} f_{18}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_2, s_2] + \\
& \quad \delta a[-2 b_{s_2} f_{18}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_1, s_2]), \\
& \{ \text{UU}[ca[f_{19}[b_{s_1}, b_{s_2}], s_2, s_2, s_1]], \text{UU}[a[g_4[b_{s_1}, b_{s_2}], s_1, s_2]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s_1} f_{19}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_2, s_2] + \\
& \quad \delta a[-2 b_{s_2} f_{19}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_1, s_2]), \\
& \{ \text{UU}[\delta aa[f_{22}[b_{s_1}, b_{s_2}], s_1, s_1, s_1, s_2]], \text{UU}[a[g_6[b_{s_1}, b_{s_2}], s_2, s_1]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s_1}^2 f_{22}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_2, s_1] + \\
& \quad \delta a[-2 b_{s_1} b_{s_2} f_{22}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_1, s_1]), \\
& \{ \text{UU}[\delta aa[f_{23}[b_{s_1}, b_{s_2}], s_1, s_2, s_1, s_2]], \text{UU}[a[g_6[b_{s_1}, b_{s_2}], s_2, s_1]] \} \rightarrow \\
& \text{UU}[\delta a[2 b_{s_1}^2 f_{23}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_2, s_2] + \\
& \quad \delta a[-2 b_{s_1} b_{s_2} f_{23}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_1, s_2] + \epsilon_1 (\delta a[2 b_{s_1}^2 f_{23}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], \\
& \quad \quad s_2, s_2] + \delta a[-2 b_{s_1} b_{s_2} f_{23}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_1, s_2])], \\
& \{ \text{UU}[\delta aa[f_{24}[b_{s_1}, b_{s_2}], s_1, s_1, s_2, s_1]], \text{UU}[a[g_4[b_{s_1}, b_{s_2}], s_1, s_2]] \} \rightarrow \\
& \text{UU}[\delta \beta[-2 b_{s_1}^2 b_{s_2} f_{24}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}]]], \\
& \{ \text{UU}[\delta aa[f_{26}[b_{s_1}, b_{s_2}], s_1, s_2, s_2, s_2]], \text{UU}[a[g_6[b_{s_1}, b_{s_2}], s_2, s_1]] \} \rightarrow \\
& \text{UU}[\delta a[-2 b_{s_1} b_{s_2} f_{26}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_2, s_2] + \\
& \quad \delta a[2 b_{s_2}^2 f_{26}[b_{s_1}, b_{s_2}] g_6[b_{s_1}, b_{s_2}], s_1, s_2]), \\
& \{ \text{UU}[\delta aa[f_{27}[b_{s_1}, b_{s_2}], s_2, s_1, s_2, s_1]], \text{UU}[a[g_4[b_{s_1}, b_{s_2}], s_1, s_2]] \} \rightarrow \\
& \text{UU}[\delta a[-4 b_{s_1} b_{s_2} f_{27}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_2, s_1] + \\
& \quad \delta a[4 b_{s_2}^2 f_{27}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_1, s_1] + \delta \beta[-4 b_{s_1} b_{s_2}^2 f_{27}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}]]], \\
& \{ \text{UU}[\delta aa[f_{28}[b_{s_1}, b_{s_2}], s_2, s_1, s_2, s_2]], \text{UU}[a[g_4[b_{s_1}, b_{s_2}], s_1, s_2]] \} \rightarrow \\
& \text{UU}[\delta a[-b_{s_1} b_{s_2} f_{28}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_2, s_2] + \\
& \quad \delta a[b_{s_2}^2 f_{28}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_1, s_2] + \epsilon_1 (\delta a[-b_{s_1} b_{s_2} f_{28}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], \\
& \quad \quad s_2, s_2] + \delta a[b_{s_2}^2 f_{28}[b_{s_1}, b_{s_2}] g_4[b_{s_1}, b_{s_2}], s_1, s_2])])].
\end{aligned}$$


```

bb[S_List] := Module[{w, bar0, t},
  w = #2 // dσ[S, bar /@ S];
  Sum[
    t = db[S[[k]]][#1, w // dσ[bar[S[[k]]], S[[k]]];
    Do[t = t // dm[bar[S[[i]]], S[[i]], S[[i]], {i, 1, k - 1}];
    Do[t = t // dm[S[[i]], bar[S[[i]], S[[i]], {i, k + 1, Length@S}];
    t,
    {k, Length@S}
  ]
] &
bb[S_++] := bb[{S}]

bb2 = bb[1, 2];
bbAS[u_, v_] := bb2[u, v] + bb2[v, u];
{u, v} = {UU[a[f[b2], 1, 1]], UU[a[1, 2, 1]]};
bbAS[u, v]
UU[c[-f[b2] b-2, 1] + δa[f[b2], -2, 1]]
UU[c[-f[b2] b-2, 1] + δa[f[b2], -2, 1]]
UU[c[-f[b2] b2, 1] + δa[f[b2], 2, 1]]
UU[ca[-b1 f'[b2], 1, 2, -1] + ca[b1 f'[b2], -1, 2, 1] +
  ca[b2 f'[b2], 1, 1, -1] + δaa[-f'[b2], 1, -1, 2, 1]]
UU[c[-b1 b2 f'[b2], 1] + ca[b2 f'[b2], 1, 1, 1] + δa[b2 f'[b2], 1, 1] + δaa[-f'[b2], 1, 1, 2, 1]]
UU[c[-b1 b2 f'[b2], 1] + ca[b2 f'[b2], 1, 1, 1] + δa[b2 f'[b2], 1, 1] + δaa[-f'[b2], 1, 1, 2, 1]]
UU[c[f[b-2] b2, 1] + δa[-f[b-2], 2, 1]]
UU[c[f[b-2] b2, 1] + δa[-f[b-2], 2, 1]]
UU[c[f[b2] b2, 1] + δa[-f[b2], 2, 1]]
UU[ca[-b2 f'[b2], 1, -1, -1] + δaa[f'[b2], -1, -1, 2, 1]]
UU[c[b1 b2 f'[b2], 1] + ca[-b2 f'[b2], 1, 1, 1] + δa[-b2 f'[b2], 1, 1] + δaa[f'[b2], 1, 1, 2, 1]]
UU[c[b1 b2 f'[b2], 1] + ca[-b2 f'[b2], 1, 1, 1] + δa[-b2 f'[b2], 1, 1] + δaa[f'[b2], 1, 1, 2, 1]]
UU[0]



---


bb3 = bb[1, 2, 3];
bbJacobi[u_, v_, w_] := bb3[u, bb3[v, w]] + bb3[v, bb3[w, u]] + bb3[w, bb3[u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → bbJacobi[##] &,
    UUBasis[3, f], UUBasis[3, g], UUBasis[3, h]
  ]],
  _ → UU[0]
]

```

ct (contract) poperties

```

Outer[
  Equal[
    ct[#1 // tm[1, 2, 1], #2],
    ct[#1, #2] // tm[1, 2, 1]
  ] &,
  UUBasis[{1, 2, 3}, {0, 1}, f], UUBasis[{0, 4}, {2, 3}, g]
] // Flatten // Union
{True}

Outer[
  {#1, #2} → Equal[
    ct[#1 // hm[1, 2, 1], #2],
    ct[#1, #2] // hm[1, 2, 1]
  ] &,
  UUBasis[{1, 2}, {0, 1, 2, 3}, f], UUBasis[{0, 4}, {4, 5}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

Outer[
  {#1, #2} → Equal[
    ct[#1, #2 // hm[2, 3, 2]],
    ct[#1, #2] // hm[2, 3, 2]
  ] &,
  UUBasis[{1, 2}, {0, 1}, f], UUBasis[{0, 3}, {2, 3, 4}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

Outer[
  {#1, #2} → Equal[
    ct[#1, #2 // tm[3, 4, 3]],
    ct[#1, #2] // tm[3, 4, 3]
  ] &,
  UUBasis[{1, 2}, {0, 1}, f], UUBasis[{0, 3, 4, 5}, {2, 3}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

```