

Pensieve header: Testing notebook for Scatter and Glow in OneCo. Continues pensieve://2016-01/, continued pensieve://2016-03/.

In the U(T)U(H) conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-02"];
<< SnG.m
```

Bases

```
UUBasis[{1, 2}, {3, 4}, f]
{UU[β[f1[b1, b2]]], UU[a[f2[b1, b2], 1, 3]], UU[δa[f3[b1, b2], 1, 3]],
UU[a[f4[b1, b2], 1, 4]], UU[δa[f5[b1, b2], 1, 4]], UU[a[f6[b1, b2], 2, 3]],
UU[δa[f7[b1, b2], 2, 3]], UU[a[f8[b1, b2], 2, 4]], UU[δa[f9[b1, b2], 2, 4]],
UU[δβ[f10[b1, b2]]], UU[c[f11[b1, b2], 3]], UU[c[f12[b1, b2], 4]],
UU[ca[f13[b1, b2], 3, 1, 3]], UU[ca[f14[b1, b2], 3, 1, 4]],
UU[ca[f15[b1, b2], 3, 2, 3]], UU[ca[f16[b1, b2], 3, 2, 4]],
UU[ca[f17[b1, b2], 4, 1, 3]], UU[ca[f18[b1, b2], 4, 1, 4]], UU[ca[f19[b1, b2], 4, 2, 3]],
UU[ca[f20[b1, b2], 4, 2, 4]], UU[δaa[f21[b1, b2], 1, 3, 1, 3]],
UU[δaa[f22[b1, b2], 1, 3, 1, 4]], UU[δaa[f23[b1, b2], 1, 4, 1, 4]],
UU[δaa[f24[b1, b2], 1, 3, 2, 3]], UU[δaa[f25[b1, b2], 1, 3, 2, 4]],
UU[δaa[f26[b1, b2], 1, 4, 2, 4]], UU[δaa[f27[b1, b2], 2, 3, 2, 3]],
UU[δaa[f28[b1, b2], 2, 3, 2, 4]], UU[δaa[f29[b1, b2], 2, 4, 2, 4]]}
```

Meta-associativity for tm

```
((# // tm[1, 2, 1] // tm[1, 3, 1]) == (# // tm[2, 3, 2] // tm[1, 2, 1])) & /@
UUBasis[{1, 2, 3, 4}, {5, 6}, f] // Union
{True}
```

Meta-associativity for hm

```
((# // hm[1, 2, 1] // hm[1, 3, 1]) == (# // hm[2, 3, 2] // hm[1, 2, 1])) & /@
UUBasis[{5, 6}, {1, 2, 3, 4}, f] // Union
{True}
```

Compatibility between tm and hts

```
tmhts[u_] :=
  (u // tm[1, 2, 1] // hts[4, 1]) - (u // hts[4, 1] // hts[4, 2] // tm[1, 2, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}
```

```
tmhts[u_] :=
  (u // tm[1, 2, 1] // hts[5, 1]) - (u // hts[5, 1] // hts[5, 2] // tm[1, 2, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}
```

```
tmhts[u_] :=
  (u // tm[2, 1, 1] // hts[4, 1]) - (u // hts[4, 2] // hts[4, 1] // tm[2, 1, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}
```

```
tmhts[u_] :=
  (u // tm[2, 1, 1] // hts[5, 1]) - (u // hts[5, 2] // hts[5, 1] // tm[2, 1, 1]);
DeleteCases[
  {#} → tmhts[#] & /@ UUBasis[{1, 2, 3}, {4, 5}, f],
  _ → UU[0]
]
{}
```

Compatibility between hm and hts

```

hmhts[u_] :=
  (u // hm[1, 2, 1] // hts[1, 4]) - (u // hts[2, 4] // hts[1, 4] // hm[1, 2, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

```

```

hmhts[u_] :=
  (u // hm[1, 2, 1] // hts[1, 5]) - (u // hts[2, 5] // hts[1, 5] // hm[1, 2, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

```

```

hmhts[u_] :=
  (u // hm[2, 1, 1] // hts[1, 4]) - (u // hts[1, 4] // hts[2, 4] // hm[2, 1, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

```

```

hmhts[u_] :=
  (u // hm[2, 1, 1] // hts[1, 5]) - (u // hts[1, 5] // hts[2, 5] // hm[2, 1, 1]);
DeleteCases[
  {#} → hmhts[#] & /@ UUBasis[{4, 5}, {1, 2, 3}, f],
  _ → UU[0]
]
{}

```

Meta-associativity for dm

```

((# // dm[1, 2, 1] // dm[1, 3, 1]) == (# // dm[2, 3, 2] // dm[1, 2, 1])) & /@
  UUBasis[4, f] // Union
{True}

```

Meta-AS and meta-Jacobi for tb

```

tbAS[u_, v_] := tb[0][u, v] + tb[0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → tbAS[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{0, 2}, {3, 4}, g]
  ]],
  _ → UU[0]
]
{}

tbJacobi[u_, v_, w_] :=
  tb[0][u, tb[0][v, w]] + tb[0][v, tb[0][w, u]] + tb[0][w, tb[0][u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → tbJacobi[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{0, 2}, {3, 4}, g], UUBasis[{0, 3}, {5, 6}, h]
  ]],
  _ → UU[0]
]
{}

```

Compatibility of tb and tm

```

tbtm[u_, v_] := Plus[
  tb[1][u, tσ[2, -2][v]] // tm[2, -2, 2],
  -tb[1][u, tσ[2, -2][v]] // tm[-2, 2, 2],
  -tb[2][u, tσ[1, -1][v]] // tm[1, -1, 1],
  tb[2][u, tσ[1, -1][v]] // tm[-1, 1, 1]
];
DeleteCases[
  Flatten[Outer[
    {##} → tbtm[##] &,
    UUBasis[{1, 2, 3}, {1, 2}, f], UUBasis[{1, 2, 4}, {3, 4}, f]
  ]],
  _ → UU[0]
]
{}

```

Meta-AS and meta-Jacobi for hb

```

hbAS[u_, v_] := hb[0][u, v] + hb[0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → hbAS[##] &,
    UUBasis[{1, 2}, {0, 1}, f], UUBasis[{3, 4}, {0, 2}, g]
  ]],
  _ → UU[0]
]
{}

hbJacobi[u_, v_, w_] :=
  hb[0][u, hb[0][v, w]] + hb[0][v, hb[0][w, u]] + hb[0][w, hb[0][u, v]];
DeleteCases[
  Flatten[Outer[
    {##} → hbJacobi[##] &,
    UUBasis[{1, 2}, {0, 1}, f], UUBasis[{3, 4}, {0, 2}, g], UUBasis[{5, 6}, {0, 3}, h]
  ]],
  _ → UU[0]
]
{}

```

Compatibility of tb and tm

```

hbhm[u_, v_] := Plus[
  hb[1][u, hσ[2, -2][v]] // hm[2, -2, 2],
  -hb[1][u, hσ[2, -2][v]] // hm[-2, 2, 2],
  -hb[2][u, hσ[1, -1][v]] // hm[1, -1, 1],
  hb[2][u, hσ[1, -1][v]] // hm[-1, 1, 1]
];
DeleteCases[
  Flatten[Outer[
    {##} → hbhm[##] &,
    UUBasis[{1, 2}, {1, 2, 3}, f], UUBasis[{3, 4}, {1, 2, 4}, g]
  ]],
  _ → UU[0]
]
{}

```

Meta-Jacobi for thb/tb and for thb/hb

```

tthJacobi[u_, v_, w_] := Plus[
  -thb[0, 0][tb[0][u, v], w] + tb[0][thb[0, 0][u, w], v] - thb[0, 0][v,
    thb[0, 0][u, w]] + tb[0][u, thb[0, 0][v, w]] + thb[0, 0][u, thb[0, 0][v, w]]
];
DeleteCases[
  Flatten[Outer[
    {##} → tthJacobi[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{0, 2}, {3, 4}, g], UUBasis[{3, 4}, {0, 5}, h]
  ]],
  _ → UU[0]
]
{}

thhJacobi[u_, v_, w_] := Plus[
  -thb[0, 0][u, hb[0][v, w]] + hb[0][thb[0, 0][u, v], w] + thb[0, 0][thb[0, 0][u, v],
    w] + hb[0][v, thb[0, 0][u, w]] - thb[0, 0][thb[0, 0][u, w], v]
];
DeleteCases[
  Flatten[Outer[
    {##} → thhJacobi[##] &,
    UUBasis[{0, 1}, {1, 2}, f], UUBasis[{2, 3}, {0, 3}, g], UUBasis[{4, 5}, {0, 4}, h]
  ]],
  _ → UU[0]
]
{}

```

Meta-AS and meta-Jacobi for db

```

dbAS[u_, v_] := db[0][u, v] + db[0][v, u];
DeleteCases[
  Flatten[Outer[
    {##} → dbAS[##] &,
    UUBasis[{0, 1, 2}, f], UUBasis[{0, 3, 4}, g]
  ]],
  _ → UU[0]
]
{}

```

```

dbJacobi[u_, v_, w_] :=
  db[0][u, db[0][v, w]] + db[0][v, db[0][w, u]] + db[0][w, db[0][u, v]];
Module[{len, bas1, bas2, bas3, test},
  len = Length[bas1 = UUBasis[{0, 1, 2}, f]];
  bas2 = UUBasis[{0, 3, 4}, g];
  bas3 = UUBasis[{0, 5, 6}, h];
  DeleteCases[
    Flatten[Table[
      test = {bas1[[i]], bas2[[j]], bas3[[k]]};
      test → dbJacobi @@ test,
      {i, 1, len - 2}, {j, i + 1, len - 1}, {k, j + 1, len}
    ]],
    _ → UU[0]
  ]
]
{}

```

AS and Jacobi for bb

```

bb4 = bb[1, 2, 3, 4];
bbAS[u_, v_] := bb4[u, v] + bb4[v, u];
DeleteCases[
  Flatten[Outer[
    {##} → bbAS[##] &,
    UUBasis[2, f], UUBasis[2, g]
  ]],
  _ → UU[0]
]
{
  {UU[a[f2[b1, b2], 1, 1]], UU[a[g6[b1, b2], 2, 1]]} →
  UU[c[b1 b2 ε1 ε17 g6[b1, b2] f2^(0,1)[b1, b2] - b1 b2 ε9 ε17 g6[b1, b2] f2^(0,1)[b1, b2], 1] +
  δa[-2 b1 ε1 ε17 g6[b1, b2] f2^(0,1)[b1, b2], 2, 1]],
  {UU[a[f4[b1, b2], 1, 2]], UU[a[g6[b1, b2], 2, 1]]} →
  UU[c[b2 ε9 ε17 f4[b1, b2] g6[b1, b2] +  $\frac{b2 \epsilon_1 \epsilon_6 \epsilon_{17} f_4[b_1, b_2] g_6[b_1, b_2]}{\epsilon_{11}}$ , 1] +
  c[-b1 ε9 ε17 f4[b1, b2] g6[b1, b2] -  $\frac{b1 \epsilon_1 \epsilon_6 \epsilon_{17} f_4[b_1, b_2] g_6[b_1, b_2]}{\epsilon_{11}}$  +
  b1 b2 ε1 ε17 g6[b1, b2] f4^(0,1)[b1, b2] - b1 b2 ε9 ε17 g6[b1, b2] f4^(0,1)[b1, b2] +
  b1 b2 ε1 ε17 f4[b1, b2] g6^(0,1)[b1, b2] - b1 b2 ε9 ε17 f4[b1, b2] g6^(0,1)[b1, b2], 2] +
  δa[-2 b1 ε1 ε17 g6[b1, b2] f4^(0,1)[b1, b2] - 2 b1 ε1 ε17 f4[b1, b2] g6^(0,1)[b1, b2], 2, 2]],
  {UU[a[f4[b1, b2], 1, 2]], UU[ca[g19[b1, b2], 2, 2, 1]]} →
  UU[c[- $\frac{b1 b2 \epsilon_1^2 \epsilon_{17} f_4[b_1, b_2] g_{19}[b_1, b_2]}{\epsilon_{11}}$  +  $\frac{b1 b2 \epsilon_1 \epsilon_9 \epsilon_{17} f_4[b_1, b_2] g_{19}[b_1, b_2]}{\epsilon_{11}}$ , 2] +

```

$$\delta a \left[\frac{2 b_1 \epsilon_1^2 \epsilon_{17} f_4 [b_1, b_2] g_{19} [b_1, b_2]}{\epsilon_{11}}, 2, 2 \right],$$

{UU[a[f₄[b₁, b₂], 1, 2]], UU[δaa[g₂₄[b₁, b₂], 1, 1, 2, 1]]} →

$$\text{UU} \left[c \left[\frac{b_1^2 b_2 \epsilon_1^2 \epsilon_{17} f_4 [b_1, b_2] g_{24} [b_1, b_2]}{\epsilon_{11}} - \frac{b_1^2 b_2 \epsilon_1 \epsilon_9 \epsilon_{17} f_4 [b_1, b_2] g_{24} [b_1, b_2]}{\epsilon_{11}}, 1 \right] + \right.$$

$$\left. \delta a \left[- \frac{2 b_1 b_2 \epsilon_1^2 \epsilon_{17} f_4 [b_1, b_2] g_{24} [b_1, b_2]}{\epsilon_{11}}, 1, 1 \right] \right],$$

{UU[a[f₆[b₁, b₂], 2, 1]], UU[a[g₂[b₁, b₂], 1, 1]]} →

$$\text{UU} \left[c \left[b_1 b_2 \epsilon_1 \epsilon_{17} f_6 [b_1, b_2] g_2^{(0,1)} [b_1, b_2] - b_1 b_2 \epsilon_9 \epsilon_{17} f_6 [b_1, b_2] g_2^{(0,1)} [b_1, b_2], 1 \right] + \right.$$

$$\left. \delta a \left[-2 b_1 \epsilon_1 \epsilon_{17} f_6 [b_1, b_2] g_2^{(0,1)} [b_1, b_2], 2, 1 \right] \right],$$

{UU[a[f₆[b₁, b₂], 2, 1]], UU[a[g₄[b₁, b₂], 1, 2]]} →

$$\text{UU} \left[c \left[b_2 \epsilon_9 \epsilon_{17} f_6 [b_1, b_2] g_4 [b_1, b_2] + \frac{b_2 \epsilon_1 \epsilon_6 \epsilon_{17} f_6 [b_1, b_2] g_4 [b_1, b_2]}{\epsilon_{11}}, 1 \right] + \right.$$

$$\left. c \left[-b_1 \epsilon_9 \epsilon_{17} f_6 [b_1, b_2] g_4 [b_1, b_2] - \frac{b_1 \epsilon_1 \epsilon_6 \epsilon_{17} f_6 [b_1, b_2] g_4 [b_1, b_2]}{\epsilon_{11}} + \right. \right.$$

$$\left. b_1 b_2 \epsilon_1 \epsilon_{17} g_4 [b_1, b_2] f_6^{(0,1)} [b_1, b_2] - b_1 b_2 \epsilon_9 \epsilon_{17} g_4 [b_1, b_2] f_6^{(0,1)} [b_1, b_2] + \right.$$

$$\left. b_1 b_2 \epsilon_1 \epsilon_{17} f_6 [b_1, b_2] g_4^{(0,1)} [b_1, b_2] - b_1 b_2 \epsilon_9 \epsilon_{17} f_6 [b_1, b_2] g_4^{(0,1)} [b_1, b_2], 2 \right] +$$

$$\left. \delta a \left[-2 b_1 \epsilon_1 \epsilon_{17} g_4 [b_1, b_2] f_6^{(0,1)} [b_1, b_2] - 2 b_1 \epsilon_1 \epsilon_{17} f_6 [b_1, b_2] g_4^{(0,1)} [b_1, b_2], 2, 2 \right] \right],$$

{UU[a[f₆[b₁, b₂], 2, 1]], UU[ca[g₁₇[b₁, b₂], 2, 1, 1]]} →

$$\text{UU} \left[c \left[- \frac{b_1 b_2 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{17} [b_1, b_2]}{\epsilon_{11}} + \frac{b_1 b_2 \epsilon_1 \epsilon_9 \epsilon_{17} f_6 [b_1, b_2] g_{17} [b_1, b_2]}{\epsilon_{11}}, 1 \right] + \right.$$

$$\left. \delta a \left[\frac{2 b_1 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{17} [b_1, b_2]}{\epsilon_{11}}, 2, 1 \right] \right],$$

{UU[a[f₆[b₁, b₂], 2, 1]], UU[ca[g₁₈[b₁, b₂], 2, 1, 2]]} →

$$\text{UU} \left[c \left[- \frac{b_1 b_2 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{18} [b_1, b_2]}{\epsilon_{11}} + \frac{b_1 b_2 \epsilon_1 \epsilon_9 \epsilon_{17} f_6 [b_1, b_2] g_{18} [b_1, b_2]}{\epsilon_{11}}, 2 \right] + \right.$$

$$\left. \delta a \left[\frac{2 b_1 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{18} [b_1, b_2]}{\epsilon_{11}}, 2, 2 \right] \right],$$

{UU[a[f₆[b₁, b₂], 2, 1]], UU[δaa[g₂₂[b₁, b₂], 1, 1, 1, 2]]} →

$$\text{UU} \left[\delta a \left[\frac{2 b_1^2 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{22} [b_1, b_2]}{\epsilon_{11}}, 2, 1 \right] + \right.$$

$$\left. \delta a \left[- \frac{2 b_1 b_2 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{22} [b_1, b_2]}{\epsilon_{11}}, 1, 1 \right] \right],$$

{UU[a[f₆[b₁, b₂], 2, 1]], UU[δaa[g₂₃[b₁, b₂], 1, 2, 1, 2]]} →

$$\text{UU} \left[c \left[- \frac{2 b_1^2 b_2 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{23} [b_1, b_2]}{\epsilon_{11}} + \frac{2 b_1^2 b_2 \epsilon_1 \epsilon_9 \epsilon_{17} f_6 [b_1, b_2] g_{23} [b_1, b_2]}{\epsilon_{11}}, 2 \right] + \right.$$

$$\left. \delta a \left[\frac{4 b_1^2 \epsilon_1^2 \epsilon_{17} f_6 [b_1, b_2] g_{23} [b_1, b_2]}{\epsilon_{11}}, 2, 2 \right] \right],$$

{UU[ca[f₁₇[b₁, b₂], 2, 1, 1]], UU[a[g₆[b₁, b₂], 2, 1]]} →

$$\text{UU} \left[c \left[- \frac{b_1 b_2 \epsilon_1^2 \epsilon_{17} f_{17} [b_1, b_2] g_6 [b_1, b_2]}{\epsilon_{11}} + \frac{b_1 b_2 \epsilon_1 \epsilon_9 \epsilon_{17} f_{17} [b_1, b_2] g_6 [b_1, b_2]}{\epsilon_{11}}, 1 \right] + \right.$$

$$\left. \delta a \left[\frac{2 b_1 \epsilon_1^2 \epsilon_{17} f_{17} [b_1, b_2] g_6 [b_1, b_2]}{\epsilon_{11}}, 2, 1 \right] \right],$$

$$\{ \text{UU}[\text{ca}[\text{f}_{18}[\text{b}_1, \text{b}_2], 2, 1, 2]], \text{UU}[\text{a}[\text{g}_6[\text{b}_1, \text{b}_2], 2, 1]] \} \rightarrow$$

$$\text{UU}\left[\text{c}\left[-\frac{\text{b}_1 \text{b}_2 \epsilon_1^2 \epsilon_{17} \text{f}_{18}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}} + \frac{\text{b}_1 \text{b}_2 \epsilon_1 \epsilon_9 \epsilon_{17} \text{f}_{18}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 2\right] + \right.$$

$$\left. \delta\text{a}\left[\frac{2 \text{b}_1 \epsilon_1^2 \epsilon_{17} \text{f}_{18}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 2, 2\right]\right],$$

$$\{ \text{UU}[\text{ca}[\text{f}_{19}[\text{b}_1, \text{b}_2], 2, 2, 1]], \text{UU}[\text{a}[\text{g}_4[\text{b}_1, \text{b}_2], 1, 2]] \} \rightarrow$$

$$\text{UU}\left[\text{c}\left[-\frac{\text{b}_1 \text{b}_2 \epsilon_1^2 \epsilon_{17} \text{f}_{19}[\text{b}_1, \text{b}_2] \text{g}_4[\text{b}_1, \text{b}_2]}{\epsilon_{11}} + \frac{\text{b}_1 \text{b}_2 \epsilon_1 \epsilon_9 \epsilon_{17} \text{f}_{19}[\text{b}_1, \text{b}_2] \text{g}_4[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 2\right] + \right.$$

$$\left. \delta\text{a}\left[\frac{2 \text{b}_1 \epsilon_1^2 \epsilon_{17} \text{f}_{19}[\text{b}_1, \text{b}_2] \text{g}_4[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 2, 2\right]\right],$$

$$\{ \text{UU}[\delta\text{aa}[\text{f}_{22}[\text{b}_1, \text{b}_2], 1, 1, 1, 2]], \text{UU}[\text{a}[\text{g}_6[\text{b}_1, \text{b}_2], 2, 1]] \} \rightarrow$$

$$\text{UU}\left[\delta\text{a}\left[\frac{2 \text{b}_1^2 \epsilon_1^2 \epsilon_{17} \text{f}_{22}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 2, 1\right] + \right.$$

$$\left. \delta\text{a}\left[-\frac{2 \text{b}_1 \text{b}_2 \epsilon_1^2 \epsilon_{17} \text{f}_{22}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 1, 1\right]\right],$$

$$\{ \text{UU}[\delta\text{aa}[\text{f}_{23}[\text{b}_1, \text{b}_2], 1, 2, 1, 2]], \text{UU}[\text{a}[\text{g}_6[\text{b}_1, \text{b}_2], 2, 1]] \} \rightarrow$$

$$\text{UU}\left[\text{c}\left[-\frac{2 \text{b}_1^2 \text{b}_2 \epsilon_1^2 \epsilon_{17} \text{f}_{23}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}} + \frac{2 \text{b}_1^2 \text{b}_2 \epsilon_1 \epsilon_9 \epsilon_{17} \text{f}_{23}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 2\right] + \right.$$

$$\left. \delta\text{a}\left[\frac{4 \text{b}_1^2 \epsilon_1^2 \epsilon_{17} \text{f}_{23}[\text{b}_1, \text{b}_2] \text{g}_6[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 2, 2\right]\right],$$

$$\{ \text{UU}[\delta\text{aa}[\text{f}_{24}[\text{b}_1, \text{b}_2], 1, 1, 2, 1]], \text{UU}[\text{a}[\text{g}_4[\text{b}_1, \text{b}_2], 1, 2]] \} \rightarrow$$

$$\text{UU}\left[\text{c}\left[\frac{\text{b}_1^2 \text{b}_2 \epsilon_1^2 \epsilon_{17} \text{f}_{24}[\text{b}_1, \text{b}_2] \text{g}_4[\text{b}_1, \text{b}_2]}{\epsilon_{11}} - \frac{\text{b}_1^2 \text{b}_2 \epsilon_1 \epsilon_9 \epsilon_{17} \text{f}_{24}[\text{b}_1, \text{b}_2] \text{g}_4[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 1\right] + \right.$$

$$\left. \delta\text{a}\left[-\frac{2 \text{b}_1 \text{b}_2 \epsilon_1^2 \epsilon_{17} \text{f}_{24}[\text{b}_1, \text{b}_2] \text{g}_4[\text{b}_1, \text{b}_2]}{\epsilon_{11}}, 1, 1\right]\right]$$

```
{
  UU[δaa[1, 1, 1, 2, 3]] // hts[3, 1] // tm[3, 1, 1] // hm[3, 1, 1],
  UU[δaa[1, 1, 1, 2, 3]] // dσ[3, -1] // hts[-1, 1] // tm[-1, 1, 1] //
  hm[-1, 1, 1]
} //
```

Column

```
UU[δaa[1, 1, 1, 2, 1]]
UU[δaa[1, 1, 1, 2, 1]]
```

```

{
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] // hts[3, 1] // tm[3, 1, 1],
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] // d $\sigma$ [3, -1],
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] // d $\sigma$ [3, -1] // hts[-1, 1],
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] // d $\sigma$ [3, -1] // hts[-1, 1] // tm[-1, 1, 1] // d $\sigma$ [-1, 3]
} // Column
UU[ $\delta_a[b_1, 2, 1] + \delta_a[-b_2, 1, 1] + \delta_{aa}[1, 1, 1, 2, 3]$ ]
UU[ca[-b1, -1, 2, 1] + ca[b1, 1, 2, -1] +
  ca[-b2, 1, 1, -1] + ca[b2, -1, 1, 1] +  $\delta_{aa}[1, 1, -1, 2, 1]$ ]
UU[ca[-b1, -1, 2, 1] + ca[b1, 1, 2, -1] + ca[-b2, 1, 1, -1] +
  ca[b2, -1, 1, 1] +  $\delta_a[-b_1, 2, 1] + \delta_a[b_2, 1, 1] + \delta_{aa}[1, 1, -1, 2, 1]$ ]
UU[ $\delta_a[-b_1, 2, 1] + \delta_a[b_2, 1, 1] + \delta_{aa}[1, 1, 1, 2, 3]$ ]

UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] // d $\sigma$ [3, -1]
UU[ca[-b1, -1, 2, 1] + ca[b1, 1, 2, -1] +
  ca[-b2, 1, 1, -1] + ca[b2, -1, 1, 1] +  $\delta_{aa}[1, 1, -1, 2, 1]$ ]

```

Bug below:

```

w = UU[ $\delta_{aa}[1, 1, 4, 2, 4]$ ];
{
  w // tm[1, 2, 1],
  w // tm[1, 2, 1] // hts[4, 1],
  w // hts[4, 1] // hts[4, 2] // tm[1, 2, 1],
  w // hts[4, 1] // hts[4, 2],
  w // hts[4, 1]
} // Column
UU[ $\delta_{aa}[1, 1, 4, 1, 4]$ ]
UU[ $\delta_a[-2 b_1, 1, 4] + \delta_{aa}[1, 1, 4, 1, 4] + \delta\beta[b_1^2]$ ]
UU[ $\delta_a[-2 b_1, 1, 4] + \delta_{aa}[1, 1, 4, 1, 4] + \delta\beta[2 b_1^2 - b_1^2 \epsilon_3]$ ]
UU[ $\delta_a[-b_1 + b_1 \epsilon_1 + b_1 \epsilon_3, 2, 4] +$ 
   $\delta_a[-b_2 - b_2 \epsilon_1 - b_2 \epsilon_3, 1, 4] + \delta_{aa}[1, 1, 4, 2, 4] + \delta\beta[2 b_1 b_2 - b_1 b_2 \epsilon_3]$ ]
UU[ $\delta_a[-b_2 \epsilon_3, 1, 4] + \delta_a[-b_1 + b_1 \epsilon_3, 2, 4] + \delta_{aa}[1, 1, 4, 2, 4] + \delta\beta[b_1 b_2]$ ]

```

```
{
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] //  $d\sigma[3, -1]$ ,
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] //  $d\sigma[3, -1]$  //  $hts[-1, 1]$ ,
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] //  $d\sigma[3, -1]$  //  $hts[-1, 1]$  //  $d\sigma[-1, 3]$ ,
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] //  $hts[3, 1]$ 
} // Column
```

```
UU[ $ca[-b_1, -1, 2, 1] + ca[b_1, 1, 2, -1] +$ 
   $ca[-b_2, 1, 1, -1] + ca[b_2, -1, 1, 1] + \delta_{aa}[1, 1, -1, 2, 1]$ ]
UU[ $ca[-b_1, -1, 2, 1] + ca[b_1, 1, 2, -1] + ca[-b_2, 1, 1, -1] +$ 
   $ca[b_2, -1, 1, 1] + \delta a[-b_1, 2, 1] + \delta a[b_2, 1, 1] + \delta_{aa}[1, 1, -1, 2, 1]$ ]
UU[ $\delta a[-b_1, 2, 1] + \delta a[b_2, 1, 1] + \delta_{aa}[1, 1, 1, 2, 3]$ ]
UU[ $\delta a[b_1 \in_3, 2, 1] + \delta a[-b_2 \in_3, 1, 1] + \delta_{aa}[1, 1, 1, 2, 3]$ ]
```

```
{
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ],
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] //  $d\sigma[3, -1]$ ,
  UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ] //  $d\sigma[3, -1]$  //  $d\sigma[-1, 3]$ 
} // Column
```

```
UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ]
UU[ $ca[-b_1, -1, 2, 1] + ca[b_1, 1, 2, -1] +$ 
   $ca[-b_2, 1, 1, -1] + ca[b_2, -1, 1, 1] + \delta_{aa}[1, 1, -1, 2, 1]$ ]
UU[ $\delta_{aa}[1, 1, 1, 2, 3]$ ]
```

```
{cand = UU[ $ca[b_2, 3, 1, 1]$ ];
  cand //  $hts[3, 1]$  ,
  cand //  $d\sigma[3, -1]$  //  $hts[-1, 1]$  //  $d\sigma[-1, 3]$ 
} // Column
```

```
UU[ $c[-b_1 b_2, 1] + ca[b_2, 3, 1, 1] + \delta a[b_2, 1, 1]$ ]
UU[ $c[-b_1 b_2, 1] + ca[b_2, 3, 1, 1] + \delta a[b_2, 1, 1]$ ]
```

```
bb3 = bb[1, 2, 3];
```

```
bbJacobi[u_, v_, w_] := bb3[u, bb3[v, w]] + bb3[v, bb3[w, u]] + bb3[w, bb3[u, v]];
```

```
DeleteCases[
```

```
  Flatten[Outer[
    {###} → bbJacobi[###] &,
    UUBasis[3, f], UUBasis[3, g], UUBasis[3, h]
  ]],
  _ → UU[0]
```

```
]
```

```
$Aborted
```

ct (contract) poperties

```

Outer[
  Equal[
    ct[#1 // tm[1, 2, 1], #2],
    ct[#1, #2] // tm[1, 2, 1]
  ] &,
  UUBasis[{1, 2, 3}, {0, 1}, f], UUBasis[{0, 4}, {2, 3}, g]
] // Flatten // Union
{True}

Outer[
  {#1, #2} → Equal[
    ct[#1 // hm[1, 2, 1], #2],
    ct[#1, #2] // hm[1, 2, 1]
  ] &,
  UUBasis[{1, 2}, {0, 1, 2, 3}, f], UUBasis[{0, 4}, {4, 5}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

Outer[
  {#1, #2} → Equal[
    ct[#1, #2 // hm[2, 3, 2]],
    ct[#1, #2] // hm[2, 3, 2]
  ] &,
  UUBasis[{1, 2}, {0, 1}, f], UUBasis[{0, 3}, {2, 3, 4}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

Outer[
  {#1, #2} → Equal[
    ct[#1, #2 // tm[3, 4, 3]],
    ct[#1, #2] // tm[3, 4, 3]
  ] &,
  UUBasis[{1, 2}, {0, 1}, f], UUBasis[{0, 3, 4, 5}, {2, 3}, g]
] // Flatten // DeleteCases[#, _ → True] &
{}

```