

Pensieve header: Scatter and Glow in OneCo. Continues pensieve://2015-12/, continued pensieve://2016-02/.

In the U(T)U(H) conventions.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2015-12"]
```

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C:\\drorbn\\AcademicPensieve\\2015-12
```

## Generalities

```
DQ[is___] := (Sort[{is}] === Union[{is}]);
OQ[is___] := OrderedQ[{is}];
(* tests for non-strict ordering. Also true when {is} is {i,i}. *)
Kδis := KroneckerDelta[is];
Simp[expr_] := Simplify[expr];
S[expr_] :=
  expr /. (λβ | λa | λδβ | λδa | λc | λca | λδaa) => MapAt[Simp, λ, 1];
AutoCollecting[λ_] := (
  λ /: λ[0, ___] = 0;
  λ /: λ[f_, r___] + λ[g_, r___] := λ[Simp[f+g], r];
  λ /: g_*λ[f_, r___] := λ[Simp[gf], r];
);
AutoCollecting /@ {β, a, δβ, c, δa, ca, δaa};
UU /: UU[x_] + UU[y_] := UU[x+y];
UU /: a_*UU[x_] := UU[Expand[a x]];
Υ[f_, j_, k_] := δa[f, j, k] - c[bj f, k];
Υa[f_, j_, k_, l_, m_] := δaa[f, j, k, l, m] - ca[bj f, k, l, m];
```

## Bases

```

UUBasis[T_List, H_List, f_] := Module[
  {ff, n = 0, h, t, h1, h2},
  ff := f_{++n} @@ Table[b_t, {t, T}];
  UU /@ Flatten@{
     $\beta$ [ff],
    Table[{a[ff, t, h],  $\delta a$ [ff, t, h]}, {t, T}, {h, H}],
     $\delta\beta$ [ff],
    Table[c[ff, h], {h, H}],
    Table[ca[ff, h1, t, h2], {h1, H}, {t, T}, {h2, H}],
    Table[ $\delta aa$ [ff, T[[i]], H[[j]], T[[k]], H[[l]],
      {k, Length@T}, {i, k}, {l, Length@H}, {j, l}]
  ] /. 1_[___]  $\rightarrow$  1
];
UUBasis[S_List, f_] := UUBasis[S, S, f];
UUBasis[n_Integer, m_Integer, f_] := UUBasis[Range@n, Range@m, f];
UUBasis[n_Integer, f_] := UUBasis[Range@n, f];

```

## $\delta aa$ relations

Switch from thth to tthh indexing? (not for the moment)

```

UU[expr_] // S := UU[S[expr] /. {
   $\delta aa$ [f_, i_, j_, k_, l_] /; !OQ[j, l]  $\Rightarrow$   $\delta aa$ [f, k, l, i, j],
   $\delta aa$ [f_, i_, j_, k_, l_] /; !OQ[i, k]  $\wedge$  DQ[j, l]  $\wedge$  OQ[j, l]  $\Rightarrow$ 
   $\delta aa$ [f, i, l, k, j] + ca[b_k f, l, i, j] +
  ca[-b_i f, l, k, j] + ca[-b_k f, j, i, l] + ca[b_i f, j, k, l]
}
]];

```

## tm, hm, hts, dm

```

UU[expr_] // tm[x_, y_, z_] := UU[S[expr] /. {
  a[f_, x, j_]  $\Rightarrow$  a[f, z, j] +  $\gamma$ [ $\partial_{b_y} f$ , z, j],
  a[f_, y, j_]  $\Rightarrow$  a[f, z, j],
   $\delta a$ [f_, x | y, j_]  $\Rightarrow$   $\delta a$ [f, z, j],
  ca[f_, i_, x | y, j_]  $\Rightarrow$  ca[f, i, z, j],
   $\delta aa$ [f_, i_, j_, k_, l_]  $\Rightarrow$   $\delta aa$ [f, i /. x | y  $\rightarrow$  z, j, k /. x | y  $\rightarrow$  z, l]
} /. b_{x|y}  $\rightarrow$  b_z]];

```

```

UU[expr_] // hm[x_, y_, z_] := S[UU[expr /. {
  a[f_, i_, x | y] => a[f, i, z],
  c[f_, x | y] => c[f, z],
  δa[f_, i_, x | y] => δa[f, i, z],
  ca[f_, y, j_, x] => ca[f, z, j, z] + γ[f, j, z],
  ca[f_, i_, j_, k_] => ca[f, i /. x | y → z, j, k /. x | y → z],
  δaa[f_, i_, y, k_, x] => δaa[f, k, z, i, z],
  δaa[f_, i_, j_, k_, l_] => δaa[f, i, j /. x | y → z, k, l /. x | y → z]
}]];

UU[expr_] // hts[y_, x_] := S[UU[expr /. {
  a[f_, i_, j_] =>
    a[f, i, j] - Kδj,yγ[∂bxf, i, y] - Kδi,xKδj,y(β[f bx] + c[f, y] - δβ[bx ∂bxf]),
  δa[f_, x, y] => δa[f, x, y] - δβ[f bx],
  ca[f_, i_, j_, k_] =>
    ca[f, i, j, k] + Kδi,yKδj,xγ[f, x, k] + Kδj,xKδk,yc[-f bx, i],
  δaa[f_, i_, j_, k_, l_] => δaa[f, i, j, k, l] + Kδi,xKδj,yδa[-bxf, k, l] +
    Kδi,xKδl,y(-δa[bkf, x, j] + δa[bxf, k, j]) +
    ε2Kδk,xKδj,y(δa[bif, x, l] - δa[bxf, i, l]) +
    Kδk,xKδl,yδa[-bxf, i, j] + Kδi,xKδj,l,yδβ[bi bkf]
}]];

dm[x_, y_, z_][expr_] := expr // hts[x, y] // tm[x, y, z] // hm[x, y, z]

```

$t\sigma, h\sigma, d\sigma$  on  $\{\beta, a, \delta\beta, c, \delta a, ca, \delta aa\}$

```

tσ[x_, y_][expr_] := expr /. bx → by /. {
  a[f_, i_, j_] => a[f, i /. x → y, j],
  δa[f_, i_, j_] => δa[f, i /. x → y, j],
  ca[f_, i_, j_, k_] => ca[f, i, j /. x → y, k],
  δaa[f_, i_, j_, k_, l_] => δa[f, i /. x → y, j, k /. x → y, l]
}

```

## tb, hb, thb, htb, db, b on $\{\beta, a, \delta\beta, c, \delta a, ca, \delta aa\}$

```

tb[x_][UU[L_], UU[R_]] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
  p[_beta | _delta beta | _c | _delta a | _ca | _delta aa, _beta | _delta beta | _c | _delta a | _ca | _delta aa] -> 0,
  p[u_beta | u_delta beta | u_c | u_delta a | u_ca | u_delta aa, v_a] :-> -p[v, u]
} /. {
  p[a[f_, x, j_], u_] :-> (u /. {
    beta[g_] :-> gamma[f delta_b_x g, x, j],
    a[g_, k_, l_] :-> gamma[a[f delta_b_x g, x, j, k, l] +
      K delta_x,k (- gamma[g delta_b_x f, k, l, x, j] + ca[f g, l, x, j] - ca[f g, j, k, l]),
    _ -> 0
  }) ,
  p[a[f_, j_, k_], a[g_, x, l_]] :-> -gamma[g delta_b_x f, x, l, j, k],
  p[_ , _] -> 0
}]]];

hb[y_][UU[L_], UU[R_]] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
  p[_beta | _delta beta, _] -> 0,
  p[_ , _beta | _delta beta] -> 0,
  p[_c | _delta a | _ca | _delta aa, _c | _delta a | _ca | _delta aa] -> 0,
  p[u_c | u_delta a | u_ca | u_delta aa, v_a] :-> -p[v, u]
} /. {
  p[a[f_, i_, y], u_] :-> (u /. {
    a[g_, j_, k_] :-> K delta_y,k (a[b_j f g, i, y] - a[b_i f g, j, k]),
    c[g_, j_] :-> K delta_y,j gamma[f g, i, j],
    delta a[g_, j_, k_] :-> K delta_y,k (delta a[b_j f g, i, y] - delta a[b_i f g, j, k]),
    ca[g_, j_, k_, l_] :->
      K delta_y,j gamma[f g, i, j, k, l] + K delta_y,l (ca[b_k f g, j, i, y] - ca[b_i f g, j, k, l]),
    delta aa[g_, j_, k_, l_, m_] :-> K delta_y,k (delta aa[b_j f g, i, y, l, m] - delta aa[b_i f g, j,
      k, l, m]) + K delta_y,m (delta aa[b_i f g, j, k, i, y] - delta aa[b_i f g, j, k, l, m])
  }) ,
  _p -> 0
}]]];

```

## ct (contract)

```

ct::usage =
  "ct[L,R,{h,t}] contracts the head h in L with the tail t in R. ct[L,R] assumes
  that both h and t are 0. When ambiguous, L is placed below R.";
ct[L_, R_] := ct[L, R, {0, 0}];
ct[UU[L_], UU[R_], {h_, t_}] := Module[{p}, S[UU[Distribute[p[L, R]] /. {
  p[_beta | _delta beta, _] -> 0,

```

```

p[a[f_, i_, h], β[g_]] := β[f bi ((∂btg) /. bt → 0)],
p[a[f_, i_, h], a[g_, t, j_]] := a[f (g /. bt → 0), i, j],
p[a[f_, i_, h], a[g_, j_, k_]] := a[f bi ((∂btg) /. bt → 0), j, k],
p[a[f_, i_, h], c[g_, j_]] := c[f bi ((∂btg) /. bt → 0), j],
p[a[f_, i_, h], δa[g_, t, j_]] := δa[f (g /. bt → 0), i, j],
p[a[f_, i_, h], δa[g_, j_, k_]] := δa[f bi ((∂btg) /. bt → 0), j, k],
p[a[f_, i_, h], ca[g_, k_, t, j_]] := ca[f (g /. bt → 0), k, i, j],
p[a[f_, i_, h], ca[g_, l_, j_, k_]] := ca[f bi ((∂btg) /. bt → 0), l, j, k],
p[a[f_, i_, h], δaa[g_, t, j_, t, k_]] → 0,
p[a[f_, i_, h], δaa[g_, t, j_, k_, l_]] := δaa[f (g /. bt → 0), i, j, k, l],
p[a[f_, i_, h], δaa[g_, j_, k_, t, l_]] := δaa[f (g /. bt → 0), j, k, i, l],
p[a[f_, i_, h], δaa[g_, j_, k_, l_, m_]] :=
  δaa[f bi ((∂btg) /. bt → 0), j, k, l, m],
p[a[_], _] → 0,
p[c[f_, h], β[g_]] := δβ[f ((∂btg) /. bt → 0)],
p[_c, _β] → 0,
p[c[f_, h], a[g_, t, j_]] := c[f (g /. bt → 0), j],
p[c[f_, h], a[g_, j_, k_]] := δa[f ((∂btg) /. bt → 0), j, k],
p[_c, _a] → 0,
p[_c | _δa | _ca | _δaa, _δβ | _c | _δa | _ca | _δaa] → 0,
p[δa[f_, i_, h], β[g_]] := δβ[f bi ((∂btg) /. bt → 0)],
p[δa[f_, i_, h], a[g_, t, j_]] := δa[f (g /. bt → 0), i, j],
p[δa[f_, i_, h], a[g_, j_, k_]] := δa[f bi ((∂btg) /. bt → 0), j, k],
p[_δa, _] → 0,
p[ca[_ , h, _ , h], _] → 0,
p[ca[f_, h, i_, j_], β[g_]] := δa[f ((∂btg) /. bt → 0), i, j],
p[ca[f_, i_, j_, h], β[g_]] := c[f bj ((∂btg) /. bt → 0), i],
p[ca[f_, h, i_, j_], a[g_, t, k_]] := ca[f (g /. bt → 0), k, i, j],
p[ca[f_, h, i_, j_], a[g_, k_, l_]] := δaa[f ((∂btg) /. bt → 0), i, j, k, l],
p[ca[f_, i_, j_, h], a[g_, t, k_]] := ca[f (g /. bt → 0), i, j, k],
p[ca[f_, i_, j_, h], a[g_, k_, l_]] := ca[f bj ((∂btg) /. bt → 0), i, k, l],
p[_ca, _] → 0,
p[δaa[_ , _ , h, _ , h], _] → 0,
p[δaa[f_, i_, h, j_, k_], β[g_]] := δa[f bi ((∂btg) /. bt → 0), j, k],
p[δaa[f_, i_, h, j_, k_], a[g_, t, l_]] := δaa[f (g /. bt → 0), i, l, j, k],
p[δaa[f_, i_, h, j_, k_], a[g_, l_, m_]] :=
  δaa[f bi ((∂btg) /. bt → 0), j, k, l, m],
p[δaa[f_, i_, j_, k_, h], β[g_]] := δa[f bk ((∂btg) /. bt → 0), i, j],
p[δaa[f_, i_, j_, k_, h], a[g_, t, l_]] := δaa[f (g /. bt → 0), i, j, k, l],
p[δaa[f_, i_, j_, k_, h], a[g_, l_, m_]] :=
  δaa[f bk ((∂btg) /. bt → 0), i, j, l, m],
p[_δaa, _] → 0
]]];

```