Pensieve header: Twisting V by R's and Θ's - continues pensieve://2015-10/.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2015-10"];
<< "../Projects/MetaCalculi/MetaCalculi.m"
```

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MetaCalculi` loading...
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```
T_i_ := e^b_i;
bConjugate[expr_] := expr /. b_i_ :> -b_i;
bSimplify[expr_] :=
  Assuming[b_1 > 0 ⋀ b_2 > 0 ⋀ b_3 > 0 ⋀ b_i > 0 ⋀ b_j > 0, FullSimplify[PowerExpand[expr]]];
Γ = bSimplify;
R[a_, b_, p_] := Γ[1, h_a + h_b T_a^p, {t_a, t_b}.{{1, 1 - T_a^p}, {0, T_a^p}}.{h_a, h_b}];
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{R[1, 2, 1], R[1, 2, 1/2] ** R[1, 2, 1/2]}
```

$$\left\{\begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & 1 & 1-e^{b_1} \\ s_2 & 0 & e^{b_1} \\ \Gamma & 1 & e^{b_1} \end{pmatrix}, \begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & 1 & 1-e^{b_1} \\ s_2 & 0 & e^{b_1} \\ \Gamma & 1 & e^{b_1} \end{pmatrix}\right\}$$

```
{t1 = Θ[1, 2, 1] // Γ, t2 = Θ[1, 2, 1/2] ** Θ[1, 2, 1/2] // Γ, bSimplify[t1 == t2]}
```

$$\left\{\begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & \frac{b_1+e^{\frac{1}{2}(b_1+b_2)}\,b_2}{b_1+b_2} & -\frac{\left(-1+e^{\frac{1}{2}(b_1+b_2)}\right)b_1}{b_1+b_2} \\ s_2 & -\frac{\left(-1+e^{\frac{1}{2}(b_1+b_2)}\right)b_2}{b_1+b_2} & \frac{e^{\frac{1}{2}(b_1+b_2)}\,b_1+b_2}{b_1+b_2} \\ \Gamma & \sqrt{e^{b_2}} & \sqrt{e^{b_1}} \end{pmatrix}, \begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & \frac{b_1+e^{\frac{1}{2}(b_1+b_2)}\,b_2}{b_1+b_2} & -\frac{\left(-1+e^{\frac{1}{2}(b_1+b_2)}\right)b_1}{b_1+b_2} \\ s_2 & -\frac{\left(-1+e^{\frac{1}{2}(b_1+b_2)}\right)b_2}{b_1+b_2} & \frac{e^{\frac{1}{2}(b_1+b_2)}\,b_1+b_2}{b_1+b_2} \\ \Gamma & \sqrt{e^{b_2}} & \sqrt{e^{b_1}} \end{pmatrix}, \text{True}\right\}$$

```
{t1 = Θ[1, 2, 7/12] // Γ, t2 = Θ[1, 2, 1/3] ** Θ[1, 2, 1/4] // Γ, bSimplify[t1 == t2]}
```

$$\left\{\begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & \frac{b_1+e^{\frac{7}{24}(b_1+b_2)}\,b_2}{b_1+b_2} & -\frac{\left(-1+e^{\frac{7}{24}(b_1+b_2)}\right)b_1}{b_1+b_2} \\ s_2 & -\frac{\left(-1+e^{\frac{7}{24}(b_1+b_2)}\right)b_2}{b_1+b_2} & \frac{e^{\frac{7}{24}(b_1+b_2)}\,b_1+b_2}{b_1+b_2} \\ \Gamma & \left(e^{b_2}\right)^{7/24} & \left(e^{b_1}\right)^{7/24} \end{pmatrix}, \begin{pmatrix} 1 & s_1 & s_2 \\ s_1 & \frac{b_1+e^{\frac{7}{24}(b_1+b_2)}\,b_2}{b_1+b_2} & -\frac{\left(-1+e^{\frac{7}{24}(b_1+b_2)}\right)b_1}{b_1+b_2} \\ s_2 & -\frac{\left(-1+e^{\frac{7}{24}(b_1+b_2)}\right)b_2}{b_1+b_2} & \frac{e^{\frac{7}{24}(b_1+b_2)}\,b_1+b_2}{b_1+b_2} \\ \Gamma & \left(e^{b_2}\right)^{7/24} & \left(e^{b_1}\right)^{7/24} \end{pmatrix}, \text{True}\right\}$$

```
Γ[V] // bSimplify
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$$\begin{pmatrix} \frac{2^{1/4}}{\left(\frac{\left(\coth\left[\frac{b_1}{2}\right]+\coth\left[\frac{b_2}{2}\right]\right)b_1\,b_2}{b_1+b_2}\right)^{1/4}} & s_1 & s_2 \\ s_1 & \frac{\sqrt{\left(-1+e^{b_1}\right)\left(-1+e^{b_2}\right)\left(-1+e^{b_1+b_2}\right)}\;b_1+e^{\frac{b_1}{2}}\left(-1+e^{b_2}\right)\sqrt{b_1\,b_2\,(b_1+b_2)}}{\sqrt{\left(-1+e^{b_1}\right)\left(-1+e^{b_2}\right)\left(-1+e^{b_1+b_2}\right)}\;(b_1+b_2)} & \frac{b_1-e^{\frac{b_1}{2}+b_2}\sqrt{\frac{\left(-1+e^{b_1}\right)b_1\,b_2\,(b_1+b_2)}{\left(-1+e^{b_2}\right)\left(-1+e^{b_1+b_2}\right)}}}{b_1+b_2} \\ s_2 & \frac{\sqrt{\left(-1+e^{b_1}\right)\left(-1+e^{b_2}\right)\left(-1+e^{b_1+b_2}\right)}\;b_2-e^{\frac{b_1}{2}}\left(-1+e^{b_2}\right)\sqrt{b_1\,b_2\,(b_1+b_2)}}{\sqrt{\left(-1+e^{b_1}\right)\left(-1+e^{b_2}\right)\left(-1+e^{b_1+b_2}\right)}\;(b_1+b_2)} & \frac{b_2+e^{\frac{b_1}{2}+b_2}\sqrt{\frac{\left(-1+e^{b_1}\right)b_1\,b_2\,(b_1+b_2)}{\left(-1+e^{b_2}\right)\left(-1+e^{b_1+b_2}\right)}}}{b_1+b_2} \\ \Gamma & 1 & e^{\frac{b_1}{2}} \end{pmatrix}$$

```
R[1, 2, -1/2] ** Γ[V] // bSimplify
```

$$\begin{pmatrix} \frac{2^{1/4}}{\left(\frac{\left(\coth\left[\frac{b_1}{2}\right]+\coth\left[\frac{b_2}{2}\right]\right) b_1 b_2}{b_1+b_2}\right)^{1/4}} & s_1 \\ s_1 & \frac{e^{\frac{b_1}{2}} (-1+e^{b_2}) \sqrt{b_1 b_2}\ (b_1+b_2)+b_1 \sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})\ (b_1+b_2)}}{\sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})}\ (b_1+b_2)^{3/2}} \quad \frac{-\left(-1+e^{\frac{b_1}{2}}\right)\left(1+e^{\frac{b_1}{2}+b_2}\right)\sqrt{b_1 b_2}}{\sqrt{(-1+e^{b} \\ s_2 & \frac{-e^{\frac{b_1}{2}} (-1+e^{b_2}) \sqrt{b_1 b_2}\ (b_1+b_2)+b_2 \sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})\ (b_1+b_2)}}{\sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})}\ (b_1+b_2)^{3/2}} \quad \frac{\left(-1+e^{\frac{b_1}{2}}\right)\left(1+e^{\frac{b_1}{2}+b_2}\right)\sqrt{b_1 b_2}}{\sqrt{(-1+e^{b} \\ \Gamma & 1 \end{pmatrix}$$

```
Γ[V] ** R[1, 2, -1/2] // bSimplify
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$$\begin{pmatrix} \frac{2^{1/4}}{\left(\frac{\left(\coth\left[\frac{b_1}{2}\right]+\coth\left[\frac{b_2}{2}\right]\right) b_1 b_2}{b_1+b_2}\right)^{1/4}} & s_1 \\ s_1 & \frac{e^{-\frac{b_1}{2}}\left(e^{\frac{b_1}{2}} (-1+e^{b_2}) \sqrt{b_1 b_2}\ (b_1+b_2)+\sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})\ (b_1+b_2)}\ \left(-b_2+e^{\frac{b_1}{2}}\ (b_1+b_2)\right)\right)}{\sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})}\ (b_1+b_2)^{3/2}} \quad \frac{e^{-\frac{b_1}{2}}\left(-}{} \\ s_2 & \frac{e^{-\frac{b_1}{2}}\left(-e^{\frac{b_1}{2}} (-1+e^{b_2}) \sqrt{b_1}\ (b_1+b_2)+\sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})\ b_2\ (b_1+b_2)}\right)}{\sqrt{\frac{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})\ (b_1+b_2)^3}{b_2}}} \\ \Gamma & 1 \end{pmatrix}$$

```
R[1, 2, 1/2] ** Γ[V] // bSimplify
```

$$\begin{pmatrix} \frac{2^{1/4}}{\left(\frac{\left(\coth\left[\frac{b_1}{2}\right]+\coth\left[\frac{b_2}{2}\right]\right) b_1 b_2}{b_1+b_2}\right)^{1/4}} & s_1 \\ s_1 & \frac{e^{\frac{b_1}{2}} (-1+e^{b_2}) \sqrt{b_1 b_2}\ (b_1+b_2)+b_1 \sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})\ (b_1+b_2)}}{\sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})}\ (b_1+b_2)^{3/2}} \quad \frac{-\left(-e^{\frac{b_1}{2}}+e^{b_1}\right)\left(-1+e^{b_2}\left(1+e^{\frac{b_1}{2}+}\right.\right.}{\sqrt{} \\ s_2 & \frac{-e^{\frac{b_1}{2}} (-1+e^{b_2}) \sqrt{b_1 b_2}\ (b_1+b_2)+b_2 \sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})\ (b_1+b_2)}}{\sqrt{(-1+e^{b_1})\ (-1+e^{b_2})\ (-1+e^{b_1+b_2})}\ (b_1+b_2)^{3/2}} \quad \frac{\left(-e^{\frac{b_1}{2}}+e^{b_1}\right)\left(-1+e^{b_2}\left(1+e^{\frac{b_1}{2}+}\right.\right.}{\sqrt{} \\ \Gamma & 1 \end{pmatrix}$$

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Table[V ** Θ[1, 2, p] // Γ, {p, {-1, -1/2, -1/4, 0, 1/4, 1/2, 1}}]
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$$\left\{\begin{array}{ll} \frac{\left(\frac{-1+e^{b_1}}{\text{Log}\left[e^{b_1}\right]}\right)^{1/4}\left(\frac{-1+e^{b_2}}{\text{Log}\left[e^{b_2}\right]}\right)^{1/4}}{\left(\frac{-1+e^{b_1+b_2}}{\text{Log}\left[e^{b_1+b_2}\right]}\right)^{1/4}} & s_1 \\ s_1 & \frac{e^{-\frac{b_2}{2}}\left(\sqrt{(-1+e^{b_2})\,b_1^3\,b_2}+\sqrt{(-1+e^{b_2})\,b_1\,b_2^3}+e^{\frac{b_2}{2}}\,b_1\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}\right)}{\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)^3}} \quad \frac{-e^{\frac{b_2}{2}}\sqrt{(-1+e^{b_1})\,b_1\,b} \\ s_2 & \frac{e^{-\frac{b_2}{2}}\left(-\sqrt{(-1+e^{b_2})\,b_1^3\,b_2}-\sqrt{(-1+e^{b_2})\,b_1\,b_2^3}+e^{\frac{b_2}{2}}\,b_2\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}\right)}{\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)^3}} \quad \frac{e^{\frac{b_2}{2}}\sqrt{(-1+e^{b_1})\,b_1\,b_2} \\ \Gamma & \frac{1}{\sqrt{e^{b_2}}} \end{array}\right.$$

$$\left(\begin{array}{ll} \frac{\left(\frac{-1+e^{b_1}}{\text{Log}\left[e^{b_1}\right]}\right)^{1/4}\left(\frac{-1+e^{b_2}}{\text{Log}\left[e^{b_2}\right]}\right)^{1/4}}{\left(\frac{-1+e^{b_1+b_2}}{\text{Log}\left[e^{b_1+b_2}\right]}\right)^{1/4}} & s_1 \qquad\qquad s_2 \\ s_1 & \frac{b_1}{b_1+b_2}+\frac{e^{\frac{b_1}{2}}\sqrt{(-1+e^{b_2})\,b_1\,b_2}}{\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}} \quad \frac{b_1}{b_1+b_2}-\frac{e^{\frac{b_1}{2}+b_2}\sqrt{(-1+e^{b_1})\,b_1\,b_2}}{\sqrt{(-1+e^{b_2})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}} \\ s_2 & \frac{b_2}{b_1+b_2}-\frac{e^{\frac{b_1}{2}}\sqrt{(-1+e^{b_2})\,b_1\,b_2}}{\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}} \quad \frac{b_2}{b_1+b_2}+\frac{e^{\frac{b_1}{2}+b_2}\sqrt{(-1+e^{b_1})\,b_1\,b_2}}{\sqrt{(-1+e^{b_2})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}} \\ \Gamma & 1 \qquad\qquad \sqrt{e^{b_1}} \end{array}\right),$$

$$\left\{\begin{array}{ll} \frac{\left(\frac{-1+e^{b_1}}{\text{Log}\left[e^{b_1}\right]}\right)^{1/4}\left(\frac{-1+e^{b_2}}{\text{Log}\left[e^{b_2}\right]}\right)^{1/4}}{\left(\frac{-1+e^{b_1+b_2}}{\text{Log}\left[e^{b_1+b_2}\right]}\right)^{1/4}} & s_1 \\ s_1 & \frac{e^{\frac{1}{8}(5\,b_1+b_2)}\sqrt{(-1+e^{b_2})\,b_1\,b_2}\;(b_1+b_2)+b_1\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}}{\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})}\;(b_1+b_2)^{3/2}} \quad \frac{-e^{\frac{5\,b_1}{8}+\frac{9\,b_2}{8}}\sqrt{(-1+e^{b_1})\,b_1\,b_2}\;(b_1} \\ s_2 & \frac{-e^{\frac{1}{8}(5\,b_1+b_2)}\sqrt{(-1+e^{b_2})\,b_1\,b_2}\;(b_1+b_2)+b_2\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})\,(b_1+b_2)}}{\sqrt{(-1+e^{b_1})\,(-1+e^{b_1+b_2})}\;(b_1+b_2)^{3/2}} \quad \frac{e^{\frac{5\,b_1}{8}+\frac{9\,b_2}{8}}\sqrt{(-1+e^{b_1})\,b_1\,b_2}\;(b_1} \\ \Gamma & \left(e^{b_2}\right)^{1/8} \end{array}\right.$$