xtx handout on 151102-09:40

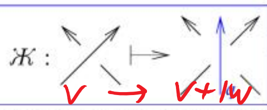
Monday, November 2, 2015 9:37 AM

Crossing the Crossings

Abstract. The subject will be very close to Manturov's representation of $v\mathcal{B}_n$ into $\mathrm{Aut}(FG_{n+1})$ — I'll describe how I think about it in terms of a very simple minded map $\mathcal{K}$ from n-component v-tangles to $(n+1)$-component w-tangles. It is possible that you all know this already. Possibly my talk will be very short — it will be as long as it is necessary to describe $\mathcal{K}$ and say a few more words, and if this is little, so be it.

All you need is $\mathcal{K}$... • What is its domain? • What is its target? • Why should one care?

$\mathcal{K}$:



Virtual Knots. Virtual knots are the algebraic structure underlying the Reidemeister presentation of ordinary knots, without the topology. Locally they are knot diagrams modulo the Reidemeister relations; globally, who cares? So,

vT = CA< ⤫, ✕ : - - - - .. >

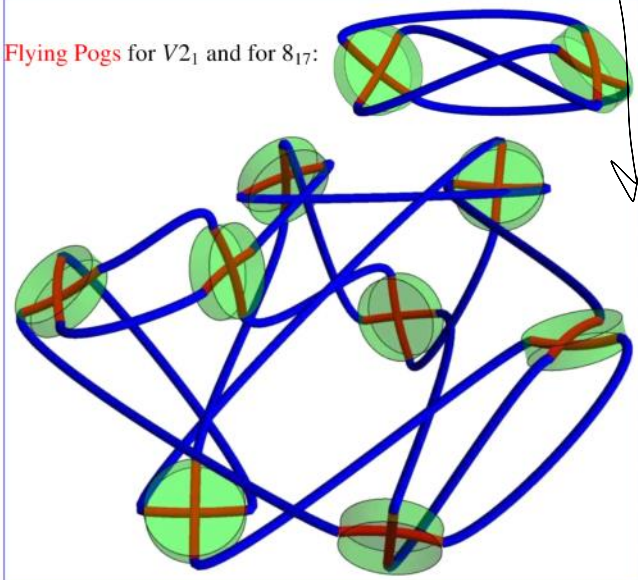
Flying pogs

Note that also

vT = PA< - - .. >

But I have a prejudice, or deeply held belief, that this is morally wrong!

Flying Pogs for $V2_1$ and for 8_{17}:



My moment of reckoning.

~~Boden and Manturov.~~ Boden's $VG_{K}(K)$

$\nwarrow\!\!\nearrow \to \begin{matrix} z = xyx^{-1} \\ w = x \end{matrix}$ $\qquad \nwarrow\!\!\nearrow \to \begin{matrix} z = x^{-1}yx \\ w = x \end{matrix}$ $\qquad \nwarrow\!\!\nearrow \to \begin{matrix} z = q^{-1}yq \\ w = qxq^{-1} \end{matrix}$

Manturov's M $v\mathcal{B}_n \to \mathrm{Aut}(F(x_1, \ldots, x_n, q))$:

$\sigma_i \mapsto \begin{cases} x_i \mapsto x_i x_{i+1} x_i^{-1} \\ x_{i+1} \mapsto x_i \end{cases}$ $\qquad \chi_i \mapsto \begin{cases} x_i \mapsto q x_{i+1} q^{-1} \\ x_{i+1} \mapsto q^{-1} x_i q \end{cases}$

Easy resolution: setting $y_i = q^i x_i$, we find that M is equivalent to

- — - -

But where does this come from?

wT = vT / OC (but not UC)

(v+1w)T = vT$_{*+1q}$ / OC on q

A

References.

[BND] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects I: Braids, Knots and the Alexander Polynomial*, ωεβ/WKO1, arXiv:1405.1956; and *II: Tangles and the Kashiwara-Vergne Problem*, ωεβ/WKO2, arXiv:1405.1955.

Add Boden, Manturov, KBH, Branden--

$\mathcal{K}_w$: vT → wT

$\mathcal{K}$: vT → (v+1w)T

$\mathcal{K}$: $v\mathcal{B}_n \to w\mathcal{B}_{n+1}$ (really, $\mathcal{B}_{nv+1w}$)

Claims 1. $\mathcal{K}$ is well-defined.

2. $\mathcal{K}$ vanishes on uT

3. $\mathcal{K}$ does not satisfy OC.

4. $VG(K) = \pi_1(\mathcal{K}(K))$

4. $M(B) = \phi(\mathcal{K}(B))$

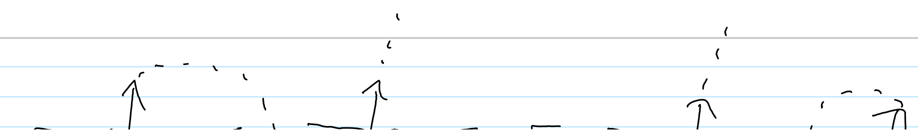
Conjecture: Brandenbursky...

Even better, can pull back any inv't from.....

Prejudices should always be re-evaluated

a rainbow box

A:



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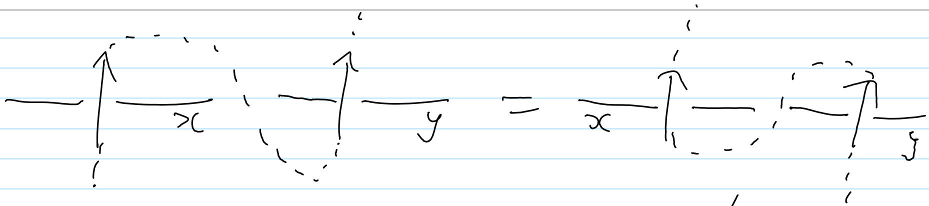

π_1 is defined on wT, Artin's rep ϕ is defined on wB_n