Pensieve header: One-Co computations in the abc presentation - testing notebook; continues pensieve://2015-08/.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2015-11"];
<< abc.m
```

Testing Jacobi and Anti-Symmetry

```
VS = False;
AS[x1_, x2_] := Module[{as},
   as = B[x1, x2] + B[x2, x1] // S;
   If[as === 0, as, {x1, x2} → as]
  ];
DeleteCases[Flatten[Outer[
   AS,
   FormalPlusBasis[3, f],
   FormalPlusBasis[3, g]
  ]], 0]
```

```
{}
```

```
AS[x1_, x2_] := Module[{as},
   as = B[x1, x2] + B[x2, x1] // S;
   If[as === 0, as, {x1, x2} → as]
  ];
DeleteCases[Flatten[Outer[
   AS,
   FormalBasis[3, f],
   FormalBasis[3, g]
  ]], 0]
```

```
{}
```

```
JacPlusErrors = DeleteCases[
  bas1 = FormalPlusBasis[4, f];
  bas2 = FormalPlusBasis[4, g];
  bas3 = FormalPlusBasis[4, h];
  Flatten[
   Table[Jacobi[bas1⟦i⟧, bas2⟦j⟧, bas3⟦k⟧],
    {i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}]
  ],
  0]
```

```
{}
```

```
VS = False;
JacErrors = DeleteCases[
  bas1 = FormalBasis[4, f];
  bas2 = FormalBasis[4, g];
  bas3 = FormalBasis[4, h];
  Flatten[
   Table[Jacobi[bas1⟦i⟧, bas2⟦j⟧, bas3⟦k⟧],
    {i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}]
  ],
  0]
{}
```

Testing U2MM and MM2U

```
U2MM /@ {a[f[b1, b2, b3], 1, 2], a[f[b1, b2, b3], 1, 1]}
```

$\{a1[f[b_1, b_2, b_3], 1, 2], a1[f[b_1, b_2, b_3], 1, 1] + c1[f[b_1, b_2, b_3] + b_1 f^{(1,0,0)}[b_1, b_2, b_3], 1] + \delta a1[-f^{(1,0,0)}[b_1, b_2, b_3], 1, 1]\}$

```
ff = f[b1, b2, b3, b4];
(# == MM2U[U2MM[#]]) & /@ {
  a[ff, 1, 2], a[ff, 1, 1], ca[ff, 1, 2, 3], ca[ff, 2, 2, 3],
  δaa[ff, 1, 2, 3, 4], δaa[ff, 1, 1, 2, 3], δaa[ff, 1, 2, 3, 3], δaa[ff, 1, 1, 2, 2]
 }
{True, True, True, True, True, True, True, True}
```

```
ff = f[b1, b2, b3, b4];
(# == (# // MM2U // U2MM)) & /@ {
  a1[ff, 1, 2], a1[ff, 1, 1], ca1[ff, 1, 2, 3], ca1[ff, 2, 2, 3], δaa1[ff, 1, 2, 3, 4],
  δaa1[ff, 1, 1, 2, 3], δaa1[ff, 1, 2, 3, 3], δaa1[ff, 1, 1, 2, 2], δaa1[ff, 3, 3, 1, 3]
 }
{True, True, True, True, True, True, True, True, True}
```

Testing $S^2 \otimes S^2$

```
VS1 = False; ff = f[b1, b2, b3, b4];
DeleteCases[
 Flatten[Table[
   δaa1[ff, i, j, k, l] →
    (δaa1[ff, i, j, k, l] // MM2U // S) - (δaa1[ff, i, j, k, l] // S // MM2U),
   {i, 4}, {j, 4}, {k, 4}, {l, 4}
  ]],
 _ → 0
]
```

{}

```
δaa1[f[b1, b2, b3, b4], 3, 3, 1, 3] // S
```

$\delta a1[f[b_1, b_2, b_3, b_4]\, b_1, 3, 3] + \delta a1[-f[b_1, b_2, b_3, b_4]\, b_3, 1, 3] + \delta aa1[f[b_1, b_2, b_3, b_4], 1, 3, 3, 3]$

```
VS1 = False; ff = f[b1, b2, b3, b4];
DeleteCases[
 Flatten[Table[
   δaa[ff, i, j, k, l] →
    (δaa[ff, i, j, k, l] // U2MM // S) - (δaa[ff, i, j, k, l] // S // U2MM),
   {i, 4}, {j, 4}, {k, 4}, {l, 4}
  ]],
 _ → 0
]
```

{}

Testing the Adjoint action

```
AutoAd[a[t, j, k]][a[1, k, j]]
```

pows computed for {a[t, j, k], a[1, k, j]}...

seq computed... a[_, j, j] is 1/20

sf1: -1 & sf2: $t^4 b_j^3 (t\, b_j)^{\#1} b_k$ &

seq computed... a[_, j, k] is 2/20

sf1: -1 & sf2: $t^4 b_j^3 (t\, b_j)^{\#1} b_k$ &

seq computed... a[_, k, j] is 3/20

sf1: 1 & sf2: $t^4 b_j^4 (t\, b_j)^{\#1}$ &

seq computed... a[_, k, k] is 4/20

sf1: 1 & sf2: $t^4 b_j^4 (t b_j)^{\#1}$ &

seq computed... c[_, j] is 5/20

sf1: $7 + 2 \#1$ & sf2: $t^4 b_j^3 (t b_j)^{\#1} b_k$ &

seq computed... c[_, k] is 6/20

sf: $t^4 b_j^3 \left(b_j (-t b_j)^{\#1} + (-t b_j)^{\#1} b_k + 7 (t b_j)^{\#1} b_k + 2 \#1 (t b_j)^{\#1} b_k\right)$ &

seq computed... ca[_, j, j, k] is 7/20

sf: $-t^4 b_j^2 (t b_j)^{\#1} (b_j - 6 b_k - 2 \#1 b_k)$ &

seq computed... ca[_, j, k, k] is 8/20

sf1: $-7 - 2 \#1$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... ca[_, k, j, j] is 9/20

sf: $-t^4 b_j^2 \left(b_j (-t b_j)^{\#1} + (-t b_j)^{\#1} b_k + 4 (t b_j)^{\#1} b_k + \#1 (t b_j)^{\#1} b_k\right)$ &

seq computed... ca[_, k, j, k] is 10/20

sf: $-t^4 b_j^2 \left(b_j (-t b_j)^{\#1} + b_j (t b_j)^{\#1} + (-t b_j)^{\#1} b_k - 2 (t b_j)^{\#1} b_k - \#1 (t b_j)^{\#1} b_k\right)$ &

seq computed... ca[_, k, k, j] is 11/20

sf1: $4 + \#1$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... ca[_, k, k, k] is 12/20

sf1: $-3 - \#1$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... δa[_, j, j] is 13/20

sf1: $-3 - \#1$ & sf2: $t^4 b_j^2 (t b_j)^{\#1} b_k$ &

seq computed... δa[_, j, k] is 14/20

sf: $-t^4 b_j^2 \left(b_j (-t b_j)^{\#1} + (-t b_j)^{\#1} b_k + 4 (t b_j)^{\#1} b_k + \#1 (t b_j)^{\#1} b_k\right)$ &

seq computed... δa[_, k, j] is 15/20

sf1: $-4 - \#1$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... δa[_, k, k] is 16/20

sf1: $-3 - \#1$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... δaa[_, j, j, j, k] is 17/20

sf: $t^4 b_j \left(b_j (-t b_j)^{\#1} + b_j (t b_j)^{\#1} + (-t b_j)^{\#1} b_k - 5 (t b_j)^{\#1} b_k - 2 \#1 (t b_j)^{\#1} b_k\right)$ &

seq computed... δaa[_, j, j, k, k] is 18/20

sf1: $2 (3 + \#1)$ & sf2: $t^4 b_j^2 (t b_j)^{\#1}$ &

seq computed... δaa[_, j, k, j, k] is 19/20

sf: $t^4 b_j \left(b_j (-t b_j)^{\#1} + b_j (t b_j)^{\#1} + (-t b_j)^{\#1} b_k - 5 (t b_j)^{\#1} b_k - 2 \#1 (t b_j)^{\#1} b_k\right)$ &

seq computed... δaa[_, j, k, k, k] is 20/20

sf1: $2 (3 + \#1)$ & sf2: $t^4 b_j^2 (t b_j)^{\#1}$ &

$$a\left[e^{t b_j}, k, j\right] + a\left[-1 + e^{t b_j}, k, k\right] + a\left[-\frac{\left(-1 + e^{t b_j}\right) b_k}{b_j}, j, j\right] + a\left[-\frac{\left(-1 + e^{t b_j}\right) b_k}{b_j}, j, k\right] +$$
$$c\left[\frac{\left(1 - e^{t b_j} + 2 e^{t b_j} t b_j\right) b_k}{b_j}, j\right] + c\left[-1 + e^{-t b_j} + \frac{e^{-t b_j}\left(1 - e^{2 t b_j} + 2 e^{2 t b_j} t b_j\right) b_k}{b_j}, k\right] +$$
$$ca\left[e^{t b_j} t, k, k, j\right] + ca\left[\frac{-1 + e^{t b_j} - 2 e^{t b_j} t b_j}{b_j}, j, k, k\right] + ca\left[\frac{-1 + e^{t b_j} - e^{t b_j} t b_j}{b_j}, k, k, k\right] +$$
$$ca\left[\frac{-2\left(-1 + e^{t b_j}\right) b_k + b_j\left(1 - e^{t b_j} + 2 e^{t b_j} t b_k\right)}{b_j^2}, j, j, k\right] +$$
$$ca\left[\frac{e^{-t b_j}\left(\left(-1 + e^{t b_j}\right) b_k + b_j\left(-1 + e^{t b_j} - e^{2 t b_j} t b_k\right)\right)}{b_j^2}, k, j, j\right] +$$
$$ca\left[\frac{1}{b_j^2} e^{-t b_j}\left(-\left(1 - 3 e^{t b_j} + 2 e^{2 t b_j}\right) b_k + b_j\left(-\left(-1 + e^{t b_j}\right)^2 + e^{2 t b_j} t b_k\right)\right), k, j, k\right] +$$
$$\delta a\left[-e^{t b_j} t, k, j\right] + \delta a\left[\frac{-1 + e^{t b_j} - e^{t b_j} t b_j}{b_j}, k, k\right] + \delta a\left[-\frac{\left(1 - e^{t b_j} + e^{t b_j} t b_j\right) b_k}{b_j^2}, j, j\right] +$$
$$\delta a\left[\frac{e^{-t b_j}\left(\left(-1 + e^{t b_j}\right) b_k + b_j\left(-1 + e^{t b_j} - e^{2 t b_j} t b_k\right)\right)}{b_j^2}, j, k\right] +$$
$$\delta aa\left[\frac{2 - 2 e^{t b_j} + 2 e^{t b_j} t b_j}{b_j^2}, j, j, k, k\right] + \delta aa\left[\frac{2 - 2 e^{t b_j} + 2 e^{t b_j} t b_j}{b_j^2}, j, k, k, k\right] +$$
$$\delta aa\left[\frac{1}{b_j^3} e^{-t b_j}\left(\left(1 - 4 e^{t b_j} + 3 e^{2 t b_j}\right) b_k + b_j\left(\left(-1 + e^{t b_j}\right)^2 - 2 e^{2 t b_j} t b_k\right)\right), j, j, j, k\right] +$$
$$\delta aa\left[\frac{1}{b_j^3} e^{-t b_j}\left(\left(1 - 4 e^{t b_j} + 3 e^{2 t b_j}\right) b_k + b_j\left(\left(-1 + e^{t b_j}\right)^2 - 2 e^{2 t b_j} t b_k\right)\right), j, k, j, k\right]$$

AutoAd - Ad tests

```
Module[{t1, t2},
   {t1 = S[AutoAd[a[t, j, k]][#]],
    S[Ad[a[t, j, k]][#] - t1]}
  ] & @ a[1, i, k]
```

pows computed for {a[t, j, k], a[1, i, k]}...

seq computed... a[_, i, k] is 1/9

sf1: $(-1)^{\#1}$ & sf2: $t^4 b_j^4 (t b_j)^{\#1}$ &

seq computed... a[_, j, k] is 2/9

sf1: $(-1)^{1+\#1}$ & sf2: $t^4 b_i b_j^3 (t b_j)^{\#1}$ &

seq computed... ca[_, k, i, k] is 3/9

sf1: $(-1)^{\#1} (-5 + 2^{4+\#1} - \#1)$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... ca[_, k, j, k] is 4/9

sf1: $-2 (-1)^{\#1} (-4 + 2^{3+\#1} - \#1)$ & sf2: $t^4 b_i b_j^2 (t b_j)^{\#1}$ &

seq computed... δa[_, i, k] is 5/9

sf1: $\frac{1}{9} 2^{-1-\#1} \left(-81 (-2)^{\#1} - 63 (-2)^{\#1} (-1)^{\#1} - 29 \times 2^{\#1} + 29 (-1)^{2 \#1} 2^{\#1} + 9 \times 2^{3+2 \#1} + 27 (-1)^{\#1} 2^{3+2 \#1} - 3 \times 2^{1+\#1} \#1 - 9 (-1)^{\#1} 2^{1+\#1} \#1 - 3 (-1)^{2 \#1} 2^{2+\#1} \#1\right)$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... δa[_, j, k] is 6/9

sf1: $-\frac{1}{9} 2^{-1-\#1} \left(-81 (-2)^{\#1} - 63 (-2)^{\#1} (-1)^{\#1} - 29 \times 2^{\#1} + 29 (-1)^{2 \#1} 2^{\#1} + 9 \times 2^{3+2 \#1} + 27 (-1)^{\#1} 2^{3+2 \#1} - 3 \times 2^{1+\#1} \#1 - 9 (-1)^{\#1} 2^{1+\#1} \#1 - 3 (-1)^{2 \#1} 2^{2+\#1} \#1\right)$ & sf2: $t^4 b_i b_j^2 (t b_j)^{\#1}$ &

seq computed... δaa[_, i, k, j, k] is 7/9

sf1: $-\frac{1}{9} 2^{-1-\#1} \left(-81 (-2)^{\#1} - 63 (-2)^{\#1} (-1)^{\#1} - 29 \times 2^{\#1} + 29 (-1)^{2 \#1} 2^{\#1} + 9 \times 2^{3+2 \#1} + 27 (-1)^{\#1} 2^{3+2 \#1} - 3 \times 2^{1+\#1} \#1 - 9 (-1)^{\#1} 2^{1+\#1} \#1 - 3 (-1)^{2 \#1} 2^{2+\#1} \#1\right)$ & sf2: $t^4 b_j^2 (t b_j)^{\#1}$ &

seq computed... δaa[_, j, k, i, k] is 8/9

sf1: $-\frac{1}{3} 2^{-1-\#1} \left(-3 (-2)^{\#1} - 9 (-2)^{\#1} (-1)^{\#1} + 13 \times 2^{\#1} + 17 (-1)^{2 \#1} 2^{\#1} - 3 \times 2^{3+2 \#1} + 3 (-1)^{\#1} 2^{3+2 \#1} - 3 (-2)^{\#1} (-1)^{\#1} \#1 + 3 \times 2^{\#1} \#1 + 3 (-1)^{2 \#1} 2^{1+\#1} \#1\right)$ & sf2: $t^4 b_j^2 (t b_j)^{\#1}$ &

seq computed... δaa[_, j, k, j, k] is 9/9

sf1: $2 (-1)^{\#1} (-4 + 2^{3+\#1} - \#1)$ & sf2: $t^4 b_i b_j (t b_j)^{\#1}$ &

$$\Big\{a\left[e^{-t b_j}, i, k\right] + a\left[\frac{\left(1 - e^{-t b_j}\right) b_i}{b_j}, j, k\right] + ca\left[\frac{e^{-2 t b_j} b_i \left(-1 + e^{2 t b_j} - 2 e^{t b_j} t b_j\right)}{b_j^2}, k, j, k\right] + ca\left[\frac{e^{-2 t b_j} \left(1 - e^{t b_j} + e^{t b_j} t b_j\right)}{b_j}, k, i, k\right] + \delta a\left[-\frac{e^{-2 t b_j} b_i \left(1 - e^{t b_j} + e^{t b_j} t b_j\right)}{b_j^2}, j, k\right] + \delta a\left[\frac{e^{-2 t b_j} \left(1 - e^{t b_j} + e^{t b_j} t b_j\right)}{b_j}, i, k\right] + \delta aa\left[-\frac{e^{-2 t b_j} b_i \left(-1 + e^{2 t b_j} - 2 e^{t b_j} t b_j\right)}{b_j^3}, j, k, j, k\right] + \delta aa\left[\frac{e^{-2 t b_j} \left(-1 + e^{t b_j} - e^{t b_j} t b_j\right)}{b_j^2}, i, k, j, k\right], 0\Big\}$$

```
AdTests[a[t, j, k]] =
  {β[f[bj, bk]], a[1, j, k], a[1, n, i], a[1, j, j], a[1, k, k], c[1, i],
   c[1, j], c[1, k], a[1, j, l], a[1, i, j], a[1, i, k], a[1, k, l], a[1, k, j]};

S[AutoAd[a[t, j, k]][#] - Ad[a[t, j, k]][#]] & /@ Take[AdTests[a[t, j, k]], All]

$Aborted
```

The semi group properties

```
Module[{t1, t2, t3, t4},
     t1 = Ad[a[t, j, k]][#] /.
       (h : (β | a | c | δa | ca | δaa))[c_, r___] :→ h[SeriesCoefficient[c, {t, 0, 1}], r];
     t2 = B[a[1, j, k], #];
     t3 = # // Ad[a[t, j, k]] // Ad[a[s, j, k]];
     t4 = # // Ad[a[t + s, j, k]];
     # → S[{t1 == t2, t3 - t4}]
   ] & /@ AdTests[a[t, j, k]] // ColumnForm

β[f[bj, bk]] → {True, 0}
a[1, j, k] → {True, 0}
a[1, n, i] → {True, 0}
a[1, j, j] → {True, 0}
a[1, k, k] → {True, 0}
c[1, i] → {True, 0}
c[1, j] → {True, 0}
c[1, k] → {True, 0}
a[1, j, l] → {True, 0}
a[1, i, j] → {True, 0}
a[1, i, k] → {True, 0}
a[1, k, l] → {True, 0}
a[1, k, j] → {True, 0}
```

```
(# → (# // R[2, 3] // S)) & /@ {
   β[b₁], β[b₂], β[b₃],
   c[1, 1], c[1, 2], c[1, 3],
   a[1, 1, 2], a[1, 1, 3], a[1, 1, 4],
   a[1, 1, 4], a[1, 2, 4], a[1, 3, 4],
   a[1/b₁, 1, 4], a[1/b₂, 2, 4], a[1/b₃, 3, 4]
  } // ColumnForm
```

$$\beta[b_1] \to \beta[b_1]$$
$$\beta[b_2] \to c\left[-1 + e^{-b_2}, 3\right] + \beta[b_2] + \delta a\left[\frac{1-e^{-b_2}}{b_2}, 2, 3\right]$$
$$\beta[b_3] \to c\left[1 - e^{-b_2}, 3\right] + \beta[b_3] + \delta a\left[\frac{-1+e^{-b_2}}{b_2}, 2, 3\right]$$
$$c[1, 1] \to c[1, 1]$$
$$c[1, 2] \to c[1, 2] + c\left[1 - e^{-b_2}, 3\right] + \delta a\left[\frac{-1+e^{-b_2}}{b_2}, 2, 3\right]$$
$$c[1, 3] \to c\left[e^{-b_2}, 3\right] + \delta a\left[\frac{1-e^{-b_2}}{b_2}, 2, 3\right]$$
$$a[1, 1, 2] \to a[1, 1, 2] + a\left[1 - e^{-b_2}, 1, 3\right] + a\left[\frac{(-1+e^{-b_2})\, b_1}{b_2}, 2, 3\right] + ca\left[-\frac{1}{2} e^{-2 b_2} \left(-1 + e^{b_2}\right), 3, 1, 3\right]$$
$$a[1, 1, 3] \to a\left[e^{-b_2}, 1, 3\right] + a\left[\frac{(1-e^{-b_2})\, b_1}{b_2}, 2, 3\right] + c\left[-\frac{e^{-b_2} b_1 \left(2-2 e^{b_2}+(1+e^{b_2})\, b_2\right)}{2 b_2}, 3\right] + ca\left[\frac{1}{2} e^{-2 b_2} \left(-1 + e^{b_2}\right.\right.$$
$$a[1, 1, 4] \to a[1, 1, 4]$$
$$a[1, 1, 4] \to a[1, 1, 4]$$
$$a[1, 2, 4] \to a[1, 2, 4] + ca[1, 4, 2, 3] + ca\left[\frac{-1+e^{-b_2}}{b_2}, 3, 2, 4\right] + \delta aa\left[-\frac{-1+e^{-b_2}+b_2}{b_2^2}, 2, 3, 2, 4\right]$$
$$a[1, 3, 4] \to a\left[e^{b_2}, 3, 4\right] + a\left[-\frac{(-1+e^{b_2})\, b_3}{b_2}, 2, 4\right] + c\left[\frac{1}{2} \left(\frac{\left(2-2 e^{b_2}+(1+e^{b_2})\, b_2\right) b_3}{b_2} - \frac{(-1+e^{b_2}) \left(2-2 e^{b_3}+(1+e^{b_3})\, b_3\right)}{-1+e^{b_3}}\right),\right.$$
$$a\left[\frac{1}{b_1}, 1, 4\right] \to a\left[\frac{1}{b_1}, 1, 4\right]$$
$$a\left[\frac{1}{b_2}, 2, 4\right] \to a\left[\frac{1}{b_2}, 2, 4\right] + ca\left[\frac{1}{b_2}, 4, 2, 3\right] + \delta aa\left[-\frac{1}{b_2^2}, 2, 3, 2, 4\right]$$
$$a\left[\frac{1}{b_3}, 3, 4\right] \to a\left[\frac{1-e^{b_2}}{b_2}, 2, 4\right] + a\left[\frac{e^{b_2}}{b_3}, 3, 4\right] + c\left[\frac{1}{2} \left(\frac{2-2 e^{b_2}+(1+e^{b_2})\, b_2}{b_2} - \frac{(-1+e^{b_2}) \left(2-2 e^{b_3}+(1+e^{b_3})\, b_3\right)}{(-1+e^{b_3})\, b_3}\right), 4\right] + ca$$

```
(# → (# // MM2U // R[2, 3] // U2MM // S)) & /@ {
   β1[b₁], β1[b₂], β1[b₃],
   c1[1, 1], c1[1, 2], c1[1, 3],
   a1[1, 1, 2], a1[1, 1, 3], a1[1, 1, 4],
   a1[1, 1, 4], a1[1, 2, 4], a1[1, 3, 4],
   a1[1/b₁, 1, 4], a1[1/b₂, 2, 4], a1[1/b₃, 3, 4]
  } // ColumnForm
```

$\beta 1[b_1] \to \beta 1[b_1]$

$\beta 1[b_2] \to c1\left[-1 + e^{-b_2}, 3\right] + \beta 1[b_2] + \delta a1\left[\frac{1-e^{-b_2}}{b_2}, 2, 3\right]$

$\beta 1[b_3] \to c1\left[1 - e^{-b_2}, 3\right] + \beta 1[b_3] + \delta a1\left[\frac{-1+e^{-b_2}}{b_2}, 2, 3\right]$

$c1[1, 1] \to c1[1, 1]$

$c1[1, 2] \to c1[1, 2] + c1\left[1 - e^{-b_2}, 3\right] + \delta a1\left[\frac{-1+e^{-b_2}}{b_2}, 2, 3\right]$

$c1[1, 3] \to c1\left[e^{-b_2}, 3\right] + \delta a1\left[\frac{1-e^{-b_2}}{b_2}, 2, 3\right]$

$a1[1, 1, 2] \to a1[1, 1, 2] + a1\left[1 - e^{-b_2}, 1, 3\right] + a1\left[\frac{(-1+e^{-b_2})\, b_1}{b_2}, 2, 3\right] + c1\left[-\frac{b_1\,(1-e^{-b_2}-b_2)}{b_2}, 3\right] + ca1\left[-\right.$

$a1[1, 1, 3] \to a1\left[e^{-b_2}, 1, 3\right] + a1\left[\frac{(1-e^{-b_2})\, b_1}{b_2}, 2, 3\right] + c1\left[-\frac{e^{-b_2}\, b_1\,(-(-1+e^{b_2})\,(-1+e^{b_3})\, b_3 + b_2\,(-(-1+e^{b_2})\,(-1+e^{b_3}) + (-}{(-1+e^{b_3})\, b_2\, b_3}\right.$

$a1[1, 1, 4] \to a1[1, 1, 4]$

$a1[1, 1, 4] \to a1[1, 1, 4]$

$a1[1, 2, 4] \to a1[1, 2, 4] + ca1[1, 4, 2, 3] + ca1\left[\frac{-1+e^{-b_2}}{b_2}, 3, 2, 4\right] + \delta aa1\left[-\frac{-1+e^{-b_2}+b_2}{b_2^2}, 2, 3, 2, 4\right]$

$a1[1, 3, 4] \to a1\left[e^{b_2}, 3, 4\right] + a1\left[-\frac{(-1+e^{b_2})\, b_3}{b_2}, 2, 4\right] + c1\left[\frac{(1-e^{b_2}+e^{b_2}\, b_2)\, b_3}{b_2}, 4\right] + ca1\left[-\frac{e^{b_3}\,(-1+e^{b_2})\, b_3}{(-1+e^{b_3})\, b_2}, 4, 2\right.$

$a1\left[\frac{1}{b_1}, 1, 4\right] \to a1\left[\frac{1}{b_1}, 1, 4\right]$

$a1\left[\frac{1}{b_2}, 2, 4\right] \to a1\left[\frac{1}{b_2}, 2, 4\right] + ca1\left[\frac{1}{b_2}, 4, 2, 3\right] + \delta aa1\left[-\frac{1}{b_2^2}, 2, 3, 2, 4\right]$

$a1\left[\frac{1}{b_3}, 3, 4\right] \to a1\left[\frac{1-e^{b_2}}{b_2}, 2, 4\right] + a1\left[\frac{e^{b_2}}{b_3}, 3, 4\right] + c1\left[\frac{1-e^{b_2}}{b_2} + \frac{-1+e^{b_2}+e^{b_2}\, b_3}{b_3}, 4\right] + ca1\left[-\frac{e^{b_3}\,(-1+e^{b_2})}{(-1+e^{b_3})\, b_2}, 4, 2, \right.$