

Pensieve header: One-Co computations in the abc presentation - testing notebook; continues pensieve://2015-08/.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2015-11"];
<< abc.m
```

Testing Jacobi and Anti-Symmetry

```
VS = False;
AS[x1_, x2_] := Module[{as},
  as = B[x1, x2] + B[x2, x1] // S;
  If[as === 0, as, {x1, x2} → as]
];
DeleteCases[Flatten[Outer[
  AS,
  FormalPlusBasis[3, f],
  FormalPlusBasis[3, g]
]], 0]
{}

AS[x1_, x2_] := Module[{as},
  as = B[x1, x2] + B[x2, x1] // S;
  If[as === 0, as, {x1, x2} → as]
];
DeleteCases[Flatten[Outer[
  AS,
  FormalBasis[3, f],
  FormalBasis[3, g]
]], 0]
{}

JacPlusErrors = DeleteCases[
  bas1 = FormalPlusBasis[4, f];
  bas2 = FormalPlusBasis[4, g];
  bas3 = FormalPlusBasis[4, h];
  Flatten[
    Table[Jacobi[bas1[[i]], bas2[[j]], bas3[[k]],
      {i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}
    ],
  ],
  0]
{}

```

```

VS = False;
JacErrors = DeleteCases[
  bas1 = FormalBasis[4, f];
  bas2 = FormalBasis[4, g];
  bas3 = FormalBasis[4, h];
  Flatten[
    Table[Jacobi[bas1[[i]], bas2[[j]], bas3[[k]],
      {i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}]
  ],
  0]
{}

```

Testing U2MM and MM2U

```

(#**1) & /@ {a[f[b1, b2, b3], 1, 2], a[f[b1, b2, b3], 1, 1]}
{a1[f[b1, b2, b3], 1, 2], a1[f[b1, b2, b3], 1, 1] +
  c1[f[b1, b2, b3] + b1 f(1,0,0)[b1, b2, b3], 1] + δa1[-f(1,0,0)[b1, b2, b3], 1, 1]}

ff = f[b1, b2, b3, b4];
(# = MM2U[#**1]) & /@ {
  a[ff, 1, 2], a[ff, 1, 1], ca[ff, 1, 2, 3], ca[ff, 2, 2, 3],
  δaa[ff, 1, 2, 3, 4], δaa[ff, 1, 1, 2, 3], δaa[ff, 1, 2, 3, 3], δaa[ff, 1, 1, 2, 2]
}
{True, True, True, True, True, True, True, True}

ff = f[b1, b2, b3, b4];
(# = MM2U[#**1]) & /@ {
  a1[ff, 1, 2], a1[ff, 1, 1], cal[ff, 1, 2, 3], cal[ff, 2, 2, 3], δaal[ff, 1, 2, 3, 4],
  δaal[ff, 1, 1, 2, 3], δaal[ff, 1, 2, 3, 3], δaal[ff, 1, 1, 2, 2], δaal[ff, 3, 3, 1, 3]
}
{True, True, True, True, True, True, True, True}

```

Testing $S^2 \otimes S^2$

```

VS1 = False; ff = f[b1, b2, b3, b4];
DeleteCases[
  Flatten[Table[
     $\delta_{aa}[ff, i, j, k, l] \rightarrow$ 
    ( $\delta_{aa}[ff, i, j, k, l] // MM2U // S$ ) - ( $\delta_{aa}[ff, i, j, k, l] // S // MM2U$ ),
    {i, 4}, {j, 4}, {k, 4}, {l, 4}
  ]],
  _ -> 0
]
{}

 $\delta_{aa}[f[b_1, b_2, b_3, b_4], 3, 3, 1, 3] // S$ 
 $\delta_{a1}[f[b_1, b_2, b_3, b_4] b_1, 3, 3] +$ 
 $\delta_{a1}[-f[b_1, b_2, b_3, b_4] b_3, 1, 3] + \delta_{aa}[f[b_1, b_2, b_3, b_4], 1, 3, 3, 3]$ 

VS1 = False; ff = f[b1, b2, b3, b4];
DeleteCases[
  Flatten[Table[
     $\delta_{aa}[ff, i, j, k, l] \rightarrow$ 
    (( $\delta_{aa}[ff, i, j, k, l] ** 1$ ) // S) - (( $\delta_{aa}[ff, i, j, k, l] // S$ ) ** 1),
    {i, 4}, {j, 4}, {k, 4}, {l, 4}
  ]],
  _ -> 0
]
{}

```

Testing the Adjoint action

```

AutoAd[a[t, j, k]][a[1, k, j]]
pows computed for {a[t, j, k], a[1, k, j]}...
seq computed... a[_ , j, j] is 1/20
sf1: -1 & sf2:  $t^4 b_j^3 (t b_j)^{n1} b_k$  &
seq computed... a[_ , j, k] is 2/20
sf1: -1 & sf2:  $t^4 b_j^3 (t b_j)^{n1} b_k$  &
seq computed... a[_ , k, j] is 3/20
sf1: 1 & sf2:  $t^4 b_j^4 (t b_j)^{n1}$  &
seq computed... a[_ , k, k] is 4/20

```

```

sf1: 1 & sf2: t^4 b_j^4 (t b_j)^#1 &
seq computed... c[_ , j] is 5/20
sf1: 7 + 2 #1 & sf2: t^4 b_j^3 (t b_j)^#1 b_k &
seq computed... c[_ , k] is 6/20
sf: t^4 b_j^3 (b_j (-t b_j)^#1 + (-t b_j)^#1 b_k + 7 (t b_j)^#1 b_k + 2 #1 (t b_j)^#1 b_k) &
seq computed... ca[_ , j, j, k] is 7/20
sf: -t^4 b_j^2 (t b_j)^#1 (b_j - 6 b_k - 2 #1 b_k) &
seq computed... ca[_ , j, k, k] is 8/20
sf1: -7 - 2 #1 & sf2: t^4 b_j^3 (t b_j)^#1 &
seq computed... ca[_ , k, j, j] is 9/20
sf: -t^4 b_j^2 (b_j (-t b_j)^#1 + (-t b_j)^#1 b_k + 4 (t b_j)^#1 b_k + #1 (t b_j)^#1 b_k) &
seq computed... ca[_ , k, j, k] is 10/20
sf: -t^4 b_j^2 (b_j (-t b_j)^#1 + b_j (t b_j)^#1 + (-t b_j)^#1 b_k - 2 (t b_j)^#1 b_k - #1 (t b_j)^#1 b_k) &
seq computed... ca[_ , k, k, j] is 11/20
sf1: 4 + #1 & sf2: t^4 b_j^3 (t b_j)^#1 &
seq computed... ca[_ , k, k, k] is 12/20
sf1: -3 - #1 & sf2: t^4 b_j^3 (t b_j)^#1 &
seq computed... da[_ , j, j] is 13/20
sf1: -3 - #1 & sf2: t^4 b_j^2 (t b_j)^#1 b_k &
seq computed... da[_ , j, k] is 14/20
sf: -t^4 b_j^2 (b_j (-t b_j)^#1 + (-t b_j)^#1 b_k + 4 (t b_j)^#1 b_k + #1 (t b_j)^#1 b_k) &
seq computed... da[_ , k, j] is 15/20
sf1: -4 - #1 & sf2: t^4 b_j^3 (t b_j)^#1 &
seq computed... da[_ , k, k] is 16/20
sf1: -3 - #1 & sf2: t^4 b_j^3 (t b_j)^#1 &
seq computed... daa[_ , j, j, j, k] is 17/20
sf: t^4 b_j (b_j (-t b_j)^#1 + b_j (t b_j)^#1 + (-t b_j)^#1 b_k - 5 (t b_j)^#1 b_k - 2 #1 (t b_j)^#1 b_k) &
seq computed... daa[_ , j, j, k, k] is 18/20
sf1: 2 (3 + #1) & sf2: t^4 b_j^2 (t b_j)^#1 &
seq computed... daa[_ , j, k, j, k] is 19/20
sf: t^4 b_j (b_j (-t b_j)^#1 + b_j (t b_j)^#1 + (-t b_j)^#1 b_k - 5 (t b_j)^#1 b_k - 2 #1 (t b_j)^#1 b_k) &
seq computed... daa[_ , j, k, k, k] is 20/20
sf1: 2 (3 + #1) & sf2: t^4 b_j^2 (t b_j)^#1 &

```

$$\begin{aligned}
& a[e^{tb_j}, k, j] + a[-1 + e^{tb_j}, k, k] + a\left[-\frac{(-1 + e^{tb_j}) b_k}{b_j}, j, j\right] + a\left[-\frac{(-1 + e^{tb_j}) b_k}{b_j}, j, k\right] + \\
& c\left[\frac{(1 - e^{tb_j} + 2 e^{tb_j} t b_j) b_k}{b_j}, j\right] + c\left[-1 + e^{-tb_j} + \frac{e^{-tb_j} (1 - e^{2tb_j} + 2 e^{2tb_j} t b_j) b_k}{b_j}, k\right] + \\
& ca[e^{tb_j} t, k, k, j] + ca\left[\frac{-1 + e^{tb_j} - 2 e^{tb_j} t b_j}{b_j}, j, k, k\right] + ca\left[\frac{-1 + e^{tb_j} - e^{tb_j} t b_j}{b_j}, k, k, k\right] + \\
& ca\left[\frac{-2(-1 + e^{tb_j}) b_k + b_j(1 - e^{tb_j} + 2 e^{tb_j} t b_k)}{b_j^2}, j, j, k\right] + \\
& ca\left[\frac{e^{-tb_j} \left((-1 + e^{tb_j}) b_k + b_j(-1 + e^{tb_j} - e^{2tb_j} t b_k) \right)}{b_j^2}, k, j, j\right] + \\
& ca\left[\frac{1}{b_j^2} e^{-tb_j} \left(-(1 - 3 e^{tb_j} + 2 e^{2tb_j}) b_k + b_j \left(-(-1 + e^{tb_j})^2 + e^{2tb_j} t b_k \right) \right), k, j, k\right] + \\
& \delta a[-e^{tb_j} t, k, j] + \delta a\left[\frac{-1 + e^{tb_j} - e^{tb_j} t b_j}{b_j}, k, k\right] + \delta a\left[-\frac{(1 - e^{tb_j} + e^{tb_j} t b_j) b_k}{b_j^2}, j, j\right] + \\
& \delta a\left[\frac{e^{-tb_j} \left((-1 + e^{tb_j}) b_k + b_j(-1 + e^{tb_j} - e^{2tb_j} t b_k) \right)}{b_j^2}, j, k\right] + \\
& \delta aa\left[\frac{2 - 2 e^{tb_j} + 2 e^{tb_j} t b_j}{b_j^2}, j, j, k, k\right] + \delta aa\left[\frac{2 - 2 e^{tb_j} + 2 e^{tb_j} t b_j}{b_j^2}, j, k, k, k\right] + \\
& \delta aa\left[\frac{1}{b_j^3} e^{-tb_j} \left((1 - 4 e^{tb_j} + 3 e^{2tb_j}) b_k + b_j \left((-1 + e^{tb_j})^2 - 2 e^{2tb_j} t b_k \right) \right), j, j, j, k\right] + \\
& \delta aa\left[\frac{1}{b_j^3} e^{-tb_j} \left((1 - 4 e^{tb_j} + 3 e^{2tb_j}) b_k + b_j \left((-1 + e^{tb_j})^2 - 2 e^{2tb_j} t b_k \right) \right), j, k, j, k\right]
\end{aligned}$$

AutoAd - Ad tests

```

Module[{t1, t2},
  {t1 = S[AutoAd[a[t, j, k]][#]],
   S[Ad[a[t, j, k]][#] - t1]}
] & @ a[1, i, k]

```

pows computed for {a[t, j, k], a[1, i, k]}...

seq computed... a[_ , i, k] is 1/9

sf1: $(-1)^{\#1}$ & sf2: $t^4 b_j^4 (t b_j)^{\#1}$ &

seq computed... a[_ , j, k] is 2/9

sf1: $(-1)^{1+\#1}$ & sf2: $t^4 b_i b_j^3 (t b_j)^{\#1}$ &

seq computed... ca[_ , k, i, k] is 3/9

sf1: $(-1)^{\#1} (-5 + 2^{4+\#1} - \#1)$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... ca[_ , k, j, k] is 4/9

sf1: $-2 (-1)^{\#1} (-4 + 2^{3+\#1} - \#1)$ & sf2: $t^4 b_i b_j^2 (t b_j)^{\#1}$ &

seq computed... $\delta a[_ , i, k]$ is 5/9

sf1: $\frac{1}{9} 2^{-1-\#1} (-81 (-2)^{\#1} - 63 (-2)^{\#1} (-1)^{\#1} - 29 \times 2^{\#1} + 29 (-1)^{2\#1} 2^{\#1} + 9 \times 2^{3+2\#1} + 27 (-1)^{\#1} 2^{3+2\#1} - 3 \times 2^{1+\#1} \#1 - 9 (-1)^{\#1} 2^{1+\#1} \#1 - 3 (-1)^{2\#1} 2^{2+\#1} \#1)$ & sf2: $t^4 b_j^3 (t b_j)^{\#1}$ &

seq computed... $\delta a[_ , j, k]$ is 6/9

sf1: $-\frac{1}{9} 2^{-1-\#1} (-81 (-2)^{\#1} - 63 (-2)^{\#1} (-1)^{\#1} - 29 \times 2^{\#1} + 29 (-1)^{2\#1} 2^{\#1} + 9 \times 2^{3+2\#1} + 27 (-1)^{\#1} 2^{3+2\#1} - 3 \times 2^{1+\#1} \#1 - 9 (-1)^{\#1} 2^{1+\#1} \#1 - 3 (-1)^{2\#1} 2^{2+\#1} \#1)$ & sf2: $t^4 b_i b_j^2 (t b_j)^{\#1}$ &

seq computed... $\delta a a[_ , i, k, j, k]$ is 7/9

sf1: $-\frac{1}{9} 2^{-1-\#1} (-81 (-2)^{\#1} - 63 (-2)^{\#1} (-1)^{\#1} - 29 \times 2^{\#1} + 29 (-1)^{2\#1} 2^{\#1} + 9 \times 2^{3+2\#1} + 27 (-1)^{\#1} 2^{3+2\#1} - 3 \times 2^{1+\#1} \#1 - 9 (-1)^{\#1} 2^{1+\#1} \#1 - 3 (-1)^{2\#1} 2^{2+\#1} \#1)$ & sf2: $t^4 b_j^2 (t b_j)^{\#1}$ &

seq computed... $\delta a a[_ , j, k, i, k]$ is 8/9

sf1: $-\frac{1}{3} 2^{-1-\#1} (-3 (-2)^{\#1} - 9 (-2)^{\#1} (-1)^{\#1} + 13 \times 2^{\#1} + 17 (-1)^{2\#1} 2^{\#1} - 3 \times 2^{3+2\#1} + 3 (-1)^{\#1} 2^{3+2\#1} - 3 (-2)^{\#1} (-1)^{\#1} \#1 + 3 \times 2^{\#1} \#1 + 3 (-1)^{2\#1} 2^{1+\#1} \#1)$ & sf2: $t^4 b_j^2 (t b_j)^{\#1}$ &

seq computed... $\delta a a[_ , j, k, j, k]$ is 9/9

sf1: $2 (-1)^{\#1} (-4 + 2^{3+\#1} - \#1)$ & sf2: $t^4 b_i b_j (t b_j)^{\#1}$ &

$$\{a[e^{-t b_j}, i, k] + a\left[\frac{(1 - e^{-t b_j}) b_i}{b_j}, j, k\right] + ca\left[\frac{e^{-2 t b_j} b_i (-1 + e^{2 t b_j} - 2 e^{t b_j} t b_j)}{b_j^2}, k, j, k\right] + ca\left[\frac{e^{-2 t b_j} (1 - e^{t b_j} + e^{t b_j} t b_j)}{b_j}, k, i, k\right] + \delta a\left[-\frac{e^{-2 t b_j} b_i (1 - e^{t b_j} + e^{t b_j} t b_j)}{b_j^2}, j, k\right] + \delta a\left[\frac{e^{-2 t b_j} (1 - e^{t b_j} + e^{t b_j} t b_j)}{b_j}, i, k\right] + \delta a a\left[-\frac{e^{-2 t b_j} b_i (-1 + e^{2 t b_j} - 2 e^{t b_j} t b_j)}{b_j^2}, j, k, j, k\right] + \delta a a\left[\frac{e^{-2 t b_j} (-1 + e^{t b_j} - e^{t b_j} t b_j)}{b_j^2}, i, k, j, k\right], 0\}$$

```

AdTests[a[t, j, k]] =
  { $\beta$ [f[bj, bk]], a[1, j, k], a[1, n, i], a[1, j, j], a[1, k, k], c[1, i],
  c[1, j], c[1, k], a[1, j, l], a[1, i, j], a[1, i, k], a[1, k, l], a[1, k, j]};

S[AutoAd[a[t, j, k]][#] - Ad[a[t, j, k]][#]] & /@ Take[AdTests[a[t, j, k]], All]
$Aborted

```

The semi group properties

```

Module[{t1, t2, t3, t4},
  t1 = Ad[a[t, j, k]][#] /.
    (h: ( $\beta$  | a | c |  $\delta$ a | ca |  $\delta$ aa)) [c_, r___]  $\Rightarrow$  h[SeriesCoefficient[c, {t, 0, 1}], r];
  t2 = B[a[1, j, k], #];
  t3 = # // Ad[a[t, j, k]] // Ad[a[s, j, k]];
  t4 = # // Ad[a[t+s, j, k]];
  #  $\rightarrow$  S[{t1 == t2, t3 - t4}]
] & /@ AdTests[a[t, j, k]] // ColumnForm

```

```

 $\beta$ [f[bj, bk]]  $\rightarrow$  {True, 0}
a[1, j, k]  $\rightarrow$  {True, 0}
a[1, n, i]  $\rightarrow$  {True, 0}
a[1, j, j]  $\rightarrow$  {True, 0}
a[1, k, k]  $\rightarrow$  {True, 0}
c[1, i]  $\rightarrow$  {True, 0}
c[1, j]  $\rightarrow$  {True, 0}
c[1, k]  $\rightarrow$  {True, 0}
a[1, j, l]  $\rightarrow$  {True, 0}
a[1, i, j]  $\rightarrow$  {True, 0}
a[1, i, k]  $\rightarrow$  {True, 0}
a[1, k, l]  $\rightarrow$  {True, 0}
a[1, k, j]  $\rightarrow$  {True, 0}

```