

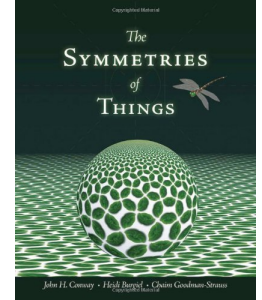
Math Union Guest Speaker, Thursday October 22 2015, University of Toronto MS 2173, 4:30PM

Abstract. My goal is to get you hooked, captured and unreleased until you find all 17 in real life, around you.

We all know know that the plane can be filled in different periodic manners: floor tiles are often square but sometimes hexagonal, bricks are often laid in an interlaced pattern, fabrics often carry interesting patterns. A little less known is that there are precisely 17 symmetry patterns for tiling the plane; not one more, not one less. It is even less known how easy these 17 are to identify in the patterns around you, how fun it is, how common some are, and how rare some others seem to be.

Gotta catch 'em all!

Reading. An excellent book on the subject is *The Symmetries of Things* by J. H. Conway, H. Burgiel, and C. Goodman-Strauss, CRC Press, 2008. Another nice text is *Classical Tessellations and Three-Manifolds* by J. M. Montesinos, Springer-Verlag, 1987.



Question. In what ways can you make \$2 change, using coins denominated $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, etc.?

Answer. $2 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{3}{4} + \frac{3}{4} + \frac{1}{2} = \frac{5}{6} + \frac{2}{3} + \frac{1}{2}$, and that's it.

Video, handout, links at drorbn.net/MUGS-1510

The Basic Features.

3

rotation only

§

rotation-reflection

M

free mirror-reflection

G

free glide-reflection

Gotta catch 'em all!

Theorem. There are precisely 17 patterns with which to tile the plane, no more, no less. They are all made of combinations of the 10 basic features, 2, 3, 4, 6, §, §, §, §, M, and G, as follows:

✓	Dror's	Conway's	crystallo-graphic	✓	Dror's	Conway's	crystallo-graphic
	2222	2222	p2		3§	3*3	p31m
	333	333	p3		2§§	2*22	cmm
	442	442	p4		22M	22*	pmg
	632	632	p6		MM	**	pm
	2§§§	*2222	pmm		MG	*o	cm
	§§§	*333	p3m1		GG	oo	pg
	44§	*442	p4m		22G	22o	pgg
	6§§	*632	p6m		\emptyset	0	p1
	4§	4*2	p4g				

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\$50 prize to the first to find a “natural”, verifiable, **333** within one hour drive of UofT!

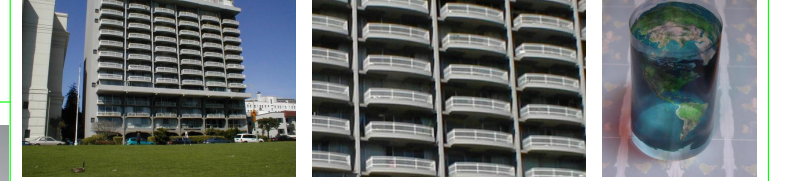
The Venus Story



The Renaissance Story

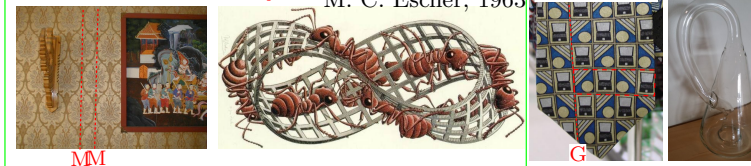


The Lake Merritt Story



The Racha Cafe Story

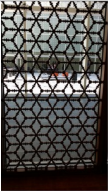
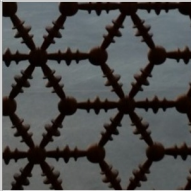



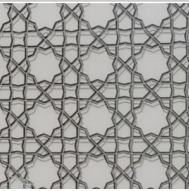
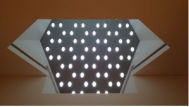
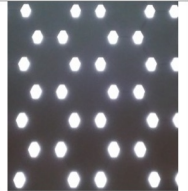
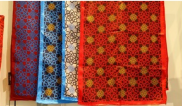








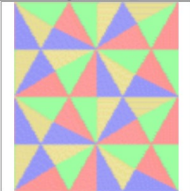
























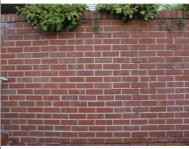



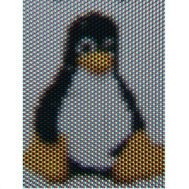
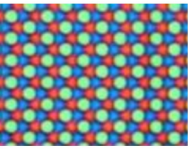











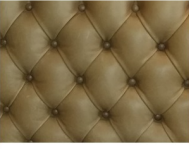




M. C. Escher, 1963



Some Further Orbifolds



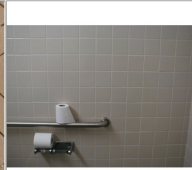
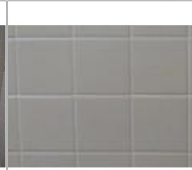
Tilings worksheet. Classify the following pictures according to the following possibilities: **2222**=2222, **333**=333, **442**=442, **632**=632, **2222**=*2222, **333**=*333, **442**=*442, **632**=*632, **42**=4*2, **33**=3*3, **222**=2*22, **22M**=22*, **MM**=*, **MG**=*o, **GG**=oo, **22G**=22o, and **0**=0 (the pictures come in {context, pattern} pairs).

							
AgaKhan-3	AgaKhan-3	AgaKhan-4	AgaKhan-4	AgaKhan-5	AgaKhan-5	AgaKhan-6	AgaKhan-6
							
AgaKhan-7	AgaKhan-7	Alhambra	Alhambra	AnteaterCage	AnteaterCage	ArchStreetFence	ArchStreetFence
							
Artificial	Artificial	AshbyTiles	AshbyTiles	BWICarpet	BWICarpet	BathroomTiles	BathroomTiles
							
BedCoverAndMitzie	BedCoverAndMitzie	BeijingBoatCeiling	BeijingBoatCeiling	BeijingStoreCeiling	BeijingStoreCeiling	BethEIQuilt	BethEIQuilt
							
BethEISidewalk	BethEISidewalk	BethlehemRoadTiles	BethlehemRoadTiles	BicycleReflector	BicycleReflector	BrakePedal	BrakePedal
							
BrickPartition	BrickPartition	BrickWall	BrickWall	BriennesNecklace	BriennesNecklace	CRTPixels	CRTPixels
							
Cactus	Cactus	CaesareaMosaic	CaesareaMosaic	ChildsDrawing	ChildsDrawing	CleaningCloth	CleaningCloth
							
ClimbingNet	ClimbingNet	CollegeStreetStarbucks	CollegeStreetStarbucks	CopenhagenTrainStation	CopenhagenTrainStation	DamascusTable	DamascusTable

Gotta!



Gotta!

							
LakeMerrittBuilding	LakeMerrittEnlarged	LalushkaFloorTiles	LalushkaFloorTiles_	Lego	Lego_	LittleItalyChair	LittleItalyChair_
							
LoyolaUniversity	LoyolaUniversity_	MSRIBathroom	MSRIBathroom_	MahaneYehudaByAssaf	MahaneYehudaByAssaf_	ManholeInLouvain	ManholeInLouvain_
							
MayasTablecloth	MayasTablecloth_	MetalGrating	MetalGrating_	MontrealSidewalk	MontrealSidewalk_	MoroccanTabletop	MoroccanTabletop_
							
NewGeometrics	NewGeometrics_	NicholasKZieveShirt	NicholasKZieveShirt_	NonSlid	NonSlid1	NonSlid_	NonSlid_
							
OldRadio	OldRadio_	OutsideWall	OutsideWall_	OxfordStreetEntryway	OxfordStreetEntryway_	PaperTowel	PaperTowel_
							
ParkingLotTiles	ParkingLotTiles_	PentagonalTiles	PentagonalTiles_	PicnicBasket	PicnicBasket_	PizzaUnoFloorTiles	PizzaUnoFloorTiles_
							
PlaygroundFence	PlaygroundFence_	PotholeCover	PotholeCover_	ProudSymbol	ProudSymbol_	PrudentialBuilding	PrudentialBuilding_
							
QueenOfHearts	QueenOfHearts_	RainbowMarketSquare	RainbowMarketSquare_	RaviOfCornell	RaviOfCornell_	RenaissanceHotel	RenaissanceHotel_
							
ReznikoffShirt	ReznikoffShirt_	RicksCafeKensington	RicksCafeKensington_	SFGGlassWindow	SFGGlassWindow_	SableSpeaker	SableSpeaker_

Gotta!

							
SafewayCart	SafewayCart_	Shredder	Shredder_	SidewalkTiles	SidewalkTiles_	SinkDrainer	SinkDrainer_
							
SisInLawShirt	SisInLawShirt_	SpadinaSubway-1	SpadinaSubway-1	SpadinaSubway-2	SpadinaSubway-2	SportsFieldFence	SportsFieldFence_
							
StClairWSubway-1	StClairWSubway-1	StClairWSubway-2	StClairWSubway-2_	StarbucksWashroom	StarbucksWashroom_	SugarPacket	SugarPacket_
							
SummerPalaceBeijing	SummerPalaceBeijing	TableMat	TableMat_	TanHall	TanHall_	ThaiWallpaper	ThaiWallpaper_
							
TheStudioBuilding	TheStudioBuilding	TieAtFrys	TieAtFrys	TieAtSFO	TieAtSFO_	TieAtSears	TieAtSears
							
TireThreads	TireThreads	TissueBox	TissueBox	Tissues	Tissues_	ToiletPaper	ToiletPaper_
							
TorontoPublicLibrary	TorontoPublicLibrary_	TransformateursInMontreal	TransformateursInMontreal_	UnderKnapp	UnderKnapp_	WalnutStreetBuilding	WalnutStreetBuilding_
							
WalthamParkTiles	WalthamParkTiles_	WaterPipes	WaterPipes_	WhiteDagoba	WhiteDagoba_	WineBottles	WineBottles_
							
WovenBasket	WovenBasket_	WovenBenchBack	WovenBenchBack_	Yeshua	Yeshua_		