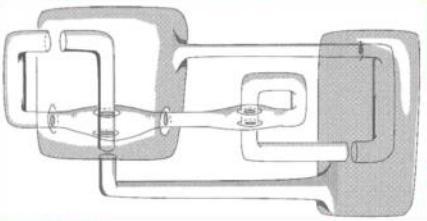
K34 handout on August 30, 2015

Sunday, August 30, 2015 4:48 PM

Dror Bar-Natan: Talks: Cornell-150925:
ω := http://www.math.toronto.edu/~drorbn/Talks/Cornell-150925

Knots in Three and Four Dimensions

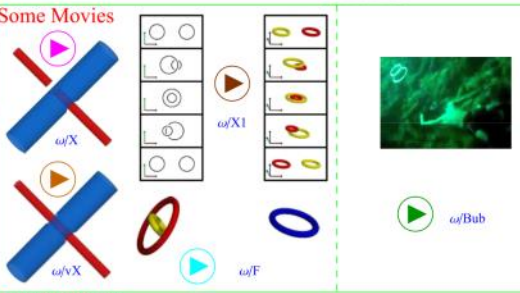
Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and if we'll still have time, we'll prove that these knots are really knotted.



ω/CS

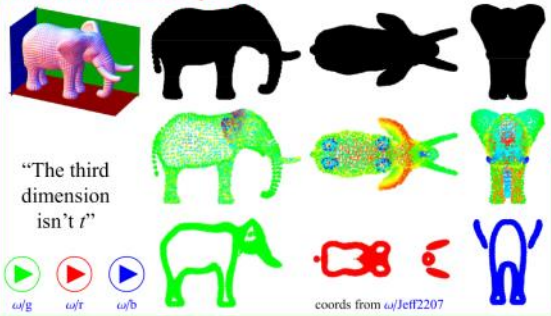
Yet another 4D Knot.

Some Movies



ω/X ω/X1 ω/vX ω/F ω/Bub

Flatlanders View an Elephant.



"The third dimension isn't *t*"

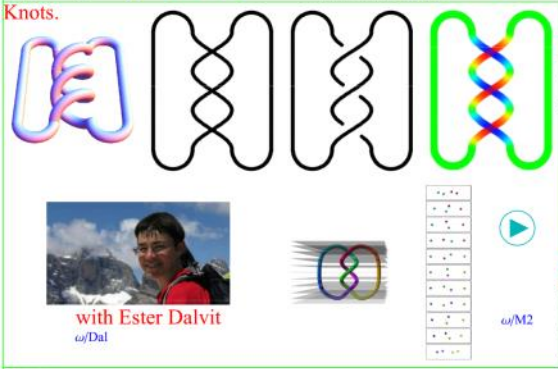
ω/g ω/t ω/b coords from ω/Jeff2207

Some Unknots

ω/U knotplot.com

Thistlethwaite's unknot Scharein's relaxation Haken's unknot

Knots.

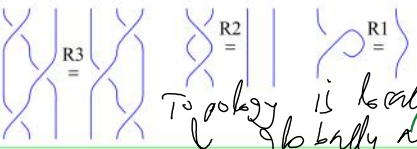


with Ester Dalvit

ω/Dal ω/M2

Reidemeister' Theorem.

Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemester moves":

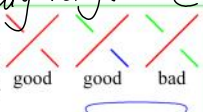


Kurt Reidemeister

Topology is locally analysis globally algebra

3-Colourings.

Colour the arcs of a broken arc diagram in RGB so that every crossing is either mono-chromatic or tri-chromatic. Let $\lambda(K)$ be the number of such 3-colourings that K has.



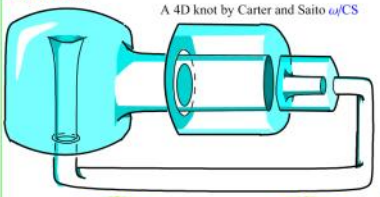
good good bad

Example. $\lambda(\bigcirc) = 3$ while $\lambda(\text{6}_3) = 9$; so $\bigcirc \neq \text{6}_3$.

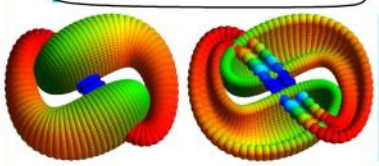
include a proof!

4D Knots.

A 4D knot by Carter and Saito ω/CS

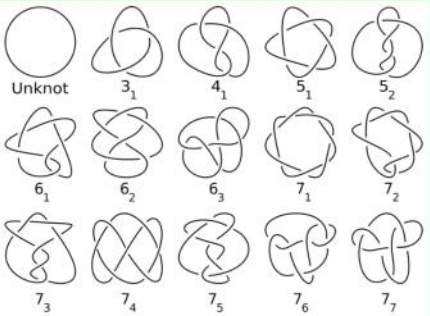


Carter, Banach, Saito



A Knot Table

There are many more!



ω/KT

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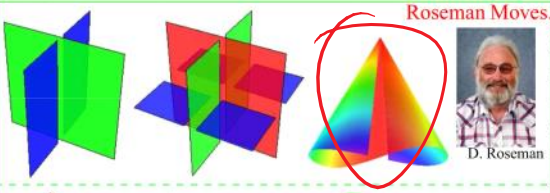
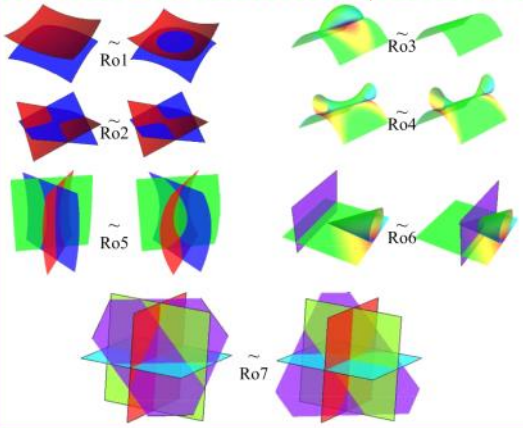
Knots in Three and Four Dimensions, 2

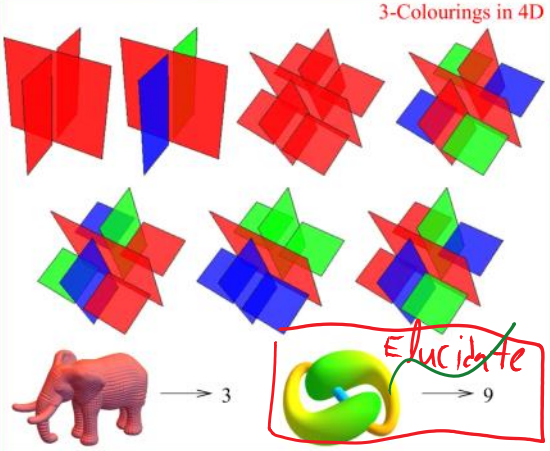
Roseman Moves.



D. Roseman

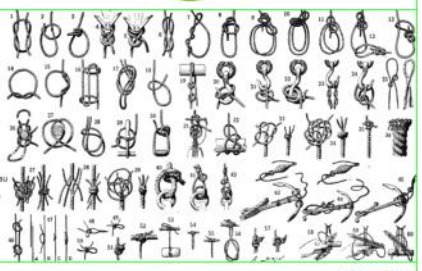


3-Colourings in 4D



Elucidate

Some knots for the practically-minded



Le Larousse pour tous : nouveau dictionnaire encyclopédique

ω/LL

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

The Knot Atlas

Some knot theory books.

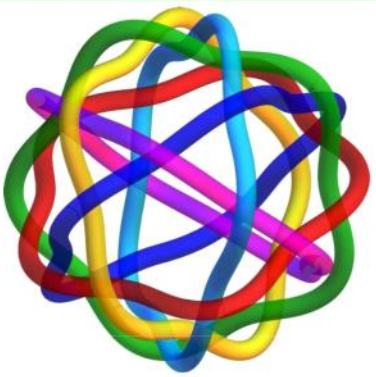
Colin C. Adams, *The Knot Book, an Elementary Introduction to the Mathematical Theory of Knots,* American Mathematical Society, 2004.

Meike Akveld and Andrew Jobbings, *Knots Unravelled, from Strings to Mathematics,* Arbelos 2011.

J. Scott Carter and Masahico Saito, *Knotted Surfaces and Their Diagrams,* American Mathematical Society, 1997.

Peter Cromwell, *Knots and Links,* Cambridge University Press, 2004.

W.B. Raymond Lickorish, *An Introduction to Knot Theory,* Springer 1997.



Banks like knots. Which knot appears twice?

clean

* Add some TT info!