Gautam on Elliptic Quantum Groups

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we restrict to sl_2, even this doesn't quite exist.

1. Felder's theory. Fix $\tau\in\mathbb{H}$, $\underset{\text{generic}}{\hbar\in\mathbb{C}^\times}$

meaning $\mathbb{Z}\hbar\cap(\Lambda_\tau=\mathbb{Z}+\tau\mathbb{Z})=\{0\}$

R-matrix:

$R(u,\lambda)\in End(\mathbb{C}^2\times\mathbb{C}^2)$ is

$$\begin{array}{c} \\ \uparrow\downarrow \\ \\ \downarrow\uparrow \end{array}\begin{bmatrix} \dfrac{\theta(u)\theta(\lambda+\hbar)}{\theta(u-\hbar)\,\theta(\lambda)} & -\dfrac{\theta(u+\lambda)\theta(\hbar)}{\theta(u-\hbar)\,\theta(\lambda)} \\ \dfrac{\theta(u-\lambda)\theta(\hbar)}{—} & \dfrac{\theta(u)\theta(\lambda-\hbar)}{—} \end{bmatrix} \quad \left(\begin{array}{c}\text{id on}\\ \uparrow\uparrow \,\&\, \downarrow\downarrow\end{array}\right)$$

where $\theta(u)$ is the unique hol. fcn s.t.

1. $\theta(u+1)=-\theta(u)$

$\theta(u+\tau)=-e^{-\pi i\tau}e^{-2\pi i}\theta(u)$

2. $\theta(u)=0\iff u\in\Lambda_\tau$

3. $\theta'(0)=1$

θ satisfies $\theta(-u)=-\theta(u)$

Properties:

1. R preserves weights:

$$h = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \circlearrowleft \mathbb{C}$$

$$[h \otimes 1 + 1 \otimes h, R(u, \lambda)] = 0$$

2 Dynamical YB eqn:

$$R^{12}(u, \lambda - \hbar wt_3) R^{13}(u+v, \lambda \quad) R^{23}(v, \lambda - \hbar wt_1 \quad)$$

$$= R^{23}(v, \lambda \quad) R^{13}(u+v, \lambda - \hbar wt_2) R^{12}(u, \lambda \quad)$$

in $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$

where wt_l = wt of l'th $\otimes$-factor

3. $R(u+1, \lambda) = R(u, \lambda) = R(u, \lambda+1)$

$$R(u+\tau, \lambda) = e^{-\pi i \hbar} (T_{\lambda - \hbar wt_2} \otimes 1) R(u, \lambda) (T_\lambda \otimes 1)^{-1}$$

where

$$T_\lambda = \begin{bmatrix} e^{-\pi i \lambda} & 0 \\ 0 & e^{\pi i \lambda} \end{bmatrix} \in SL_2(\mathbb{C})$$

Category $\mathcal{E}_{\hbar, \tau}(\mathfrak{sl}_2)$ (Felder's notation: $Rep(E_{\hbar, \tau}(\mathfrak{sl}_2))$)

Objects: $(V, L_V(u, \lambda))$ where V is

a f.d. v.s. w/ $h \in End(V)$ semi-simple
& $L_V(u,\lambda) \in End(\mathbb{C}^2 \otimes V)$ meromorphic
s.t. 1. zero weight as before.
2. YB for one R & 2 L's.
3. periodicity as for R.

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