Pensieve header: Verifying that the representation ρ is Hermitian on u-primitives.

```
n = 6;
DD = DiagonalMatrix[Table[b_i, {i, n}]];
Unprotect[ConjugateTranspose]; (
 ConjugateTranspose /: (M_)^† /; MatrixQ[M] := Transpose[M] /. {b_i_ :> -b_i};
); Protect[ConjugateTranspose];
ρ[x_Plus] := ρ /@ x;
ρ[bs_. a[j_, k_]] := bs * (Table[0, {n}, {n}] // ReplacePart[{
     {j, k} → -b_k, {k, k} → b_j
    }]);
ρ[bs_. t[j_, k_]] := ρ[bs * a[j, k] + bs * a[k, j]];
ρ[bs_. Y[i_, j_, k_]] := ρ[Expand[bs * (
     b_i a[j, k] + b_j a[k, i] + b_k a[i, j] - b_j a[i, k] - b_k a[j, i] - b_i a[k, j]
    )]];
ρ[bs_. H[i_, j_, k_, l_]] := ρ[Expand[bs * (
     b_i b_k a[j, l] - b_i b_l a[j, k] - b_j b_k a[i, l] +
      b_j b_l a[i, k] + b_i b_k a[l, j] - b_i b_l a[k, j] - b_j b_k a[l, i] + b_j b_l a[k, i]
    )]];
```

```
MatrixForm /@ {ρ[7 t[2, 3]], ρ[7 t[2, 3]]^†, DD}
```

$$\left\{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 b_3 & -7 b_3 & 0 & 0 & 0 \\ 0 & -7 b_2 & 7 b_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -7 b_3 & 7 b_2 & 0 & 0 & 0 \\ 0 & 7 b_3 & -7 b_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_6 \end{pmatrix}\right\}$$

```
MatrixForm /@ {A = ρ[t[2, 4]], A^†.DD + DD.A}
```

$$\left\{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_4 & 0 & -b_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -b_2 & 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

```
MatrixForm /@ {A = ρ[Y[2, 3, 4]], A^†.DD + DD.A}
```

$$\left\{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -b_3 b_4 & b_3 b_4 & 0 & 0 \\ 0 & b_2 b_4 & 0 & -b_2 b_4 & 0 & 0 \\ 0 & -b_2 b_3 & b_2 b_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

```
MatrixForm /@ {A = ρ[H[2, 3, 4, 5]], A^†.DD + DD.A}
```

$$\left\{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -b_3 b_4 b_5 & b_3 b_4 b_5 & 0 \\ 0 & 0 & 0 & b_2 b_4 b_5 & -b_2 b_4 b_5 & 0 \\ 0 & -b_2 b_3 b_5 & b_2 b_3 b_5 & 0 & 0 & 0 \\ 0 & b_2 b_3 b_4 & -b_2 b_3 b_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

```
MatrixForm /@ {A = ρ[H[2, 3, 3, 4]], A†.DD + DD.A}
```

$$\left\{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -b_3^2 b_4 & b_3^2 b_4 & 0 & 0 \\ 0 & -b_2 b_3 b_4 & 4 b_2 b_3 b_4 & -b_2 b_3 b_4 & 0 & 0 \\ 0 & b_2 b_3^2 & -b_2 b_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

```
MatrixForm /@ {A = ρ[H[2, 3, 2, 3]], A†.DD + DD.A}
```

$$\left\{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4 b_2 b_3^2 & 2 b_2 b_3^2 & 0 & 0 & 0 \\ 0 & 2 b_2^2 b_3 & -4 b_2^2 b_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$