Cheat Sheet Unitarity

$b_i = \log(T_i)$

σ calculus. $\sigma_1 * \sigma_2 = \sigma_1 \cup \sigma_2$, $\quad tm^{uv}_w = (T_{u,v} \to T_w)$, $\quad hm^{xy}_z(\sigma) = (\sigma\backslash\{x,y\}) \cup (z \to \sigma_x\sigma_y)$, $\quad tha^{ux} = I$, $\quad R^{\pm}_{ux} \mapsto T^{\pm 1}_u$

Gassner calculus Γ. Preserves $C_1 :=$ [col sum = 1] ($\Leftrightarrow$OC) and ✓ $C_2 := [\forall a, b,\ (T_a - 1) \mid (A_{ab} - \delta_{ab}\sigma_b)]$

$$\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\beta,\ T_a,T_b\to T_c]{m^{ab}_c} \begin{array}{c|cc} \mu\omega & c & S \\ \hline c & \gamma+\alpha\delta/\mu & \epsilon+\delta\theta/\mu \\ S & \phi+\alpha\psi/\mu & \Xi+\psi\theta/\mu \end{array}$$

$$\Theta_{ij} \underset{\Gamma}{=} \begin{array}{c|cc} 1 & i & j \\ \hline i & \frac{b_j e^{(b_i+b_j)/2}+b_i}{b_i+b_j} & \frac{b_i\left(1-e^{(b_i+b_j)/2}\right)}{b_i+b_j} \\ j & \frac{b_j\left(1-e^{(b_i+b_j)/2}\right)}{b_i+b_j} & \frac{b_i e^{(b_i+b_j)/2}+b_j}{b_i+b_j} \end{array}$$

• At $T_* = 1$, $\omega = 1$ and $A = I$.

$$R^{\pm}_{ab} \underset{\Gamma}{=} \begin{array}{c|cc} 1 & a & b \\ \hline a & 1 & 1-T_a^{\pm 1} \\ b & 0 & T_a^{\pm 1} \end{array}$$

The map (tangle $T \mapsto$ matrix A) is anti-multiplicative.

Unitarity (rough). With $\Omega(\tau) := \begin{pmatrix} (1-t_{\tau 1})^{-1} & 0 & \cdots & 0 \\ 1 & (1-t_{\tau 2})^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \ldots & (1-t_{\tau n})^{-1} \end{pmatrix}$, have $\Omega(\tau)\gamma^{-1} = \bar\gamma^T\Omega(\iota)$, or $\gamma^{-1} = \Omega(\tau)^{-1}\bar\gamma^T\Omega(\iota)$.

KV in Γ: (Represent!)

$\frac{\left(\frac{-1+T_1}{\text{Log}[T_1]}\right)^{1/4}\left(\frac{-1+T_2}{\text{Log}[T_2]}\right)^{1/4}}{\left(\frac{-1+T_1 T_2}{\text{Log}[T_1 T_2]}\right)^{1/4}}$	s_1	s_2
s_1	$\frac{\text{Log}[T_1]+\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]}$	$\frac{\text{Log}[T_1]-T_2\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]}$
s_2	$\frac{\text{Log}[T_2]-\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,T_1\,(-1+T_2)}{(-1+T_1)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]}$	$\frac{\text{Log}[T_2]+T_2\sqrt{\frac{\text{Log}[T_1]\,\text{Log}[T_2]\,\text{Log}[T_1 T_2]\,(-1+T_1)\,T_1}{(-1+T_2)\,(-1+T_1 T_2)}}}{\text{Log}[T_1 T_2]}$
Γ	1	$\sqrt{T_1}$

Unitarity Monoblog.

(150922) Is the unitarity pairing hidden in Kawazumi-Kuno?

(150921b) Is there unitarity in Cimasoni-Turaev?

(150921a) I can develop a "unitary Burau (Gassner) calculus", a functor on **PaT** (or on parenthesized bottom tangles). May follow Cimasoni-Turaev notation.

(150920) \$75 KaL question: Let $R_n = \mathbb{Q}[b_1,\ldots,b_n]$. Consider the adjoint action of the primitives $\mathcal{A}^w_{\text{prim}}(\uparrow^n)$ on $FL(n) := FL(x_1,\ldots x_n)$ via the inclusion $FL(n) \hookrightarrow \mathcal{A}^w(\uparrow^n\uparrow_\infty)$ given by $x_i \mapsto a_{i\infty}$. Reducing mod β: $[a_{ik}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$ makes R^n_n an $\mathcal{A}^w_{\text{prim}}(\uparrow^n)$-module. Restricting to $\mathcal{A}^u_{\text{prim}}(\uparrow^n)$ and regarding the b_i as imaginary, this module is formally Hermitian relative to $\langle x_i, x_j\rangle = b_i\delta_{ij}$. E.g. on $t^{jk} \in \mathfrak{t}_n$ is the matrix $A = \begin{pmatrix} b_k & -b_k \\ -b_j & b_j \end{pmatrix}$ relative to the basis $\{x_j, x_k\}$, and with $D = \begin{pmatrix} b_j & 0 \\ 0 & b_k \end{pmatrix}$, we have $\bar{A}^T D + DA = 0$. Whence cometh? Verification: pensieve://2015-09/nb/RhoHermitian.pdf.

(150916) The composition $S \times FL(S) \to \mathfrak{sder}_S \to FL(S)^S$ does not descend mod β.

(140609) Determine the image of $\mathcal{P}^u \to \mathcal{P}^w \to \mathcal{P}^w/\beta \subset M_{S\times S}(R_S)$.

(150915) A unitarity / hermitian property in $\mathcal{A}^u_{\text{exp}}$ / $\mathcal{A}^u_{\text{prim}}$?

(150906b) A Γ meaning for the u full- and half-twist belt trick?

(150906a) In u, (lazy mirror)=(radical mirror). Γ meaning?

In http://drorbn.net/AP/2012-05/beta5.1/:

Pensieve Header: Simplifying the Exact Solution.

```
SetDirectory[
  "C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.1"];
<< betaCalculus.m
Unprotect[C];
βSimplify = FullSimplify;

{V = B[ω[c1, c2], α[c1, c2] t[1] h[1] + β[c1, c2] t[1] h[2] +
    γ[c1, c2] t[2] h[1] + δ[c1, c2] t[2] h[2]],
 C = B[κ[c1], 0]}

{( ω[c1, c2]   h[1]         h[2]
   t[1]        α[c1, c2]    β[c1, c2]
   t[2]        γ[c1, c2]    δ[c1, c2] ), ( κ[c1]
                                            t[1] )}

ν[x_] := Sqrt[Sinh[x/2] / (x/2)];

κ[x_] := ν[x]^(-1/2);
ω[c1, c2] = κ[c1 + c2] / (κ[c1] κ[c2]);
γ[c1, c2] = (ν[c2] - ν[c1] ν[c1 + c2]) / ((c1 + c2) ν[c1] ν[c1 + c2]);
δ[c1, c2] = e^(c1/2)/c2 - (ν[c1 + c2] e^(c1+c2) ν[c1] c1) / ((-1 + e^(c1+c2)) ν[c2] c2) - 1/(c1 + c2);
α[c1, c2] = (-c2/c1) γ[c1, c2];
β[c1, c2] = (1/c1) (e^(c1/2) - c2 δ[c1, c2] - 1);

{V1, C1, sol1} = Get["ExactSolution-120528.m"];
FullSimplify[V == V1 && C == C1]

True
```