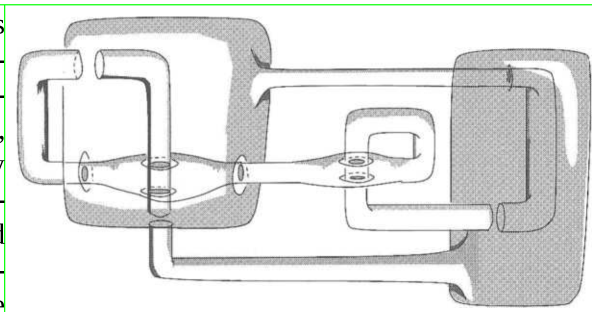
Knots in Three and Four Dimensions

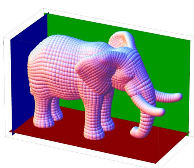
Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and if we'll still have time, we'll prove that these knots are really knotted.

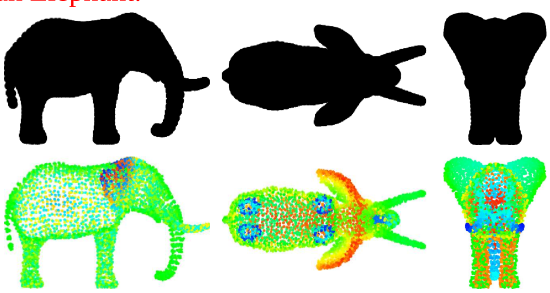


ω/CS

Yet another 4D Knot.

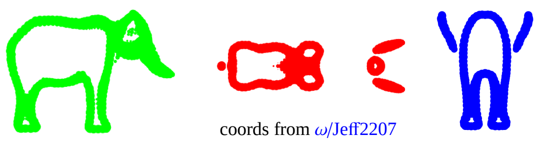
Flatlanders View an Elephant.





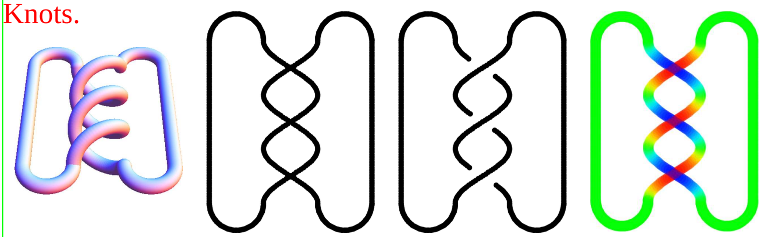
"The third dimension isn't t"

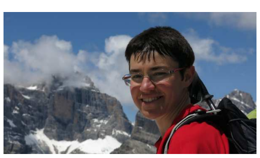
ω/g ω/r ω/b



coords from ω/Jeff2207

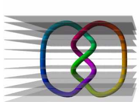
Knots.

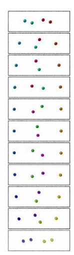




with Ester Dalvit

ω/Dal

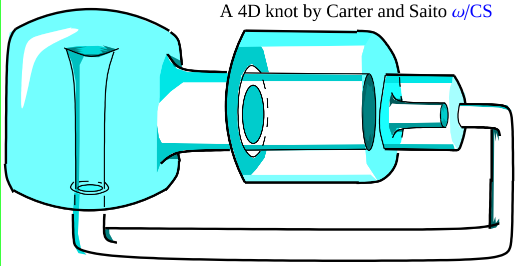




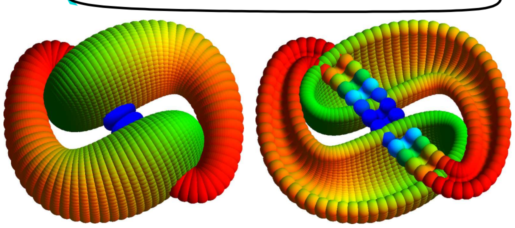
ω/M2

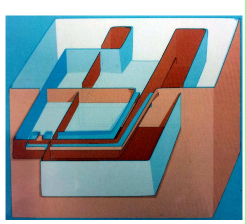
4D Knots.

A 4D knot by Carter and Saito ω/CS

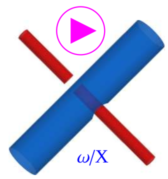


Carter, Banach, Saito

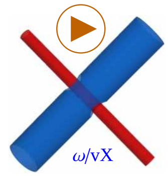




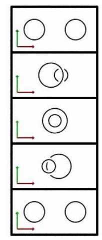
Some Movies

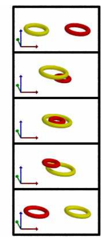


ω/X



ω/vX



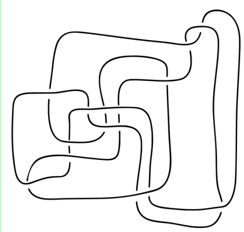


ω/X1

ω/F

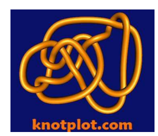
ω/Bub

Some Unknots



Thistlethwaite's unknot

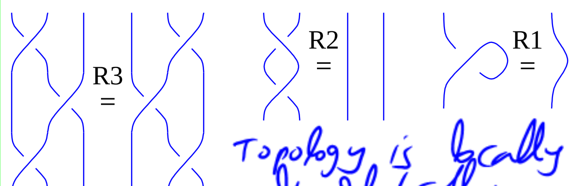
ω/U



Scharein's relaxation

Haken's unknot

Reidemeister' Theorem. Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemester moves":

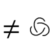


Kurt Reidemeister

Topology is locally analysis
& globally algebra

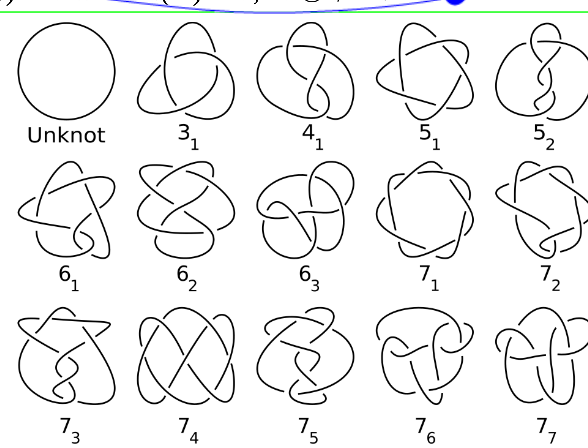
Include a proof!

3-Colourings. Colour the arcs of a broken arc diagram in RGB so that every crossing is either mono-chromatic or tri-chromatic. Let $\lambda(K)$ be the number of such 3-colourings that K has.

Example. $\lambda(\bigcirc) = 3$ while $\lambda($$) = 9$; so $\bigcirc \neq$ .

A Knot Table

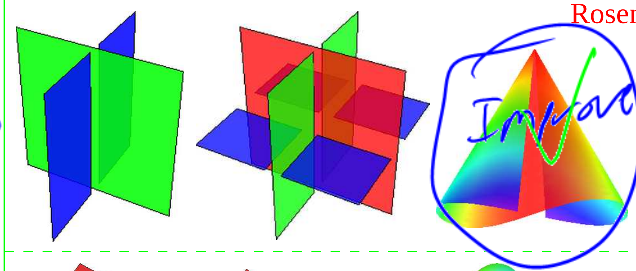
There are many more!



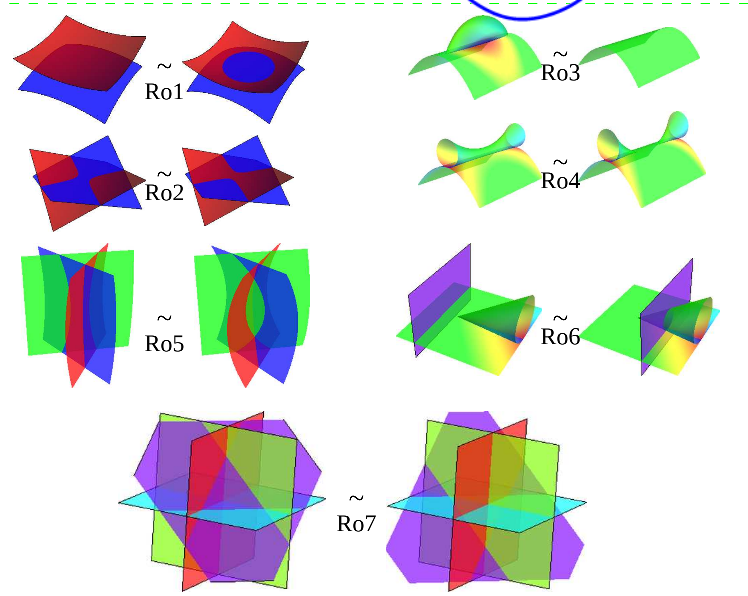
ω/KT

Knots in Three and Four Dimensions, 2

Roseman Moves.




D. Roseman



Some knot theory books.

Colin C. Adams, *The Knot Book, an Elementary Introduction to the Mathematical Theory of Knots*, American Mathematical Society, 2004.

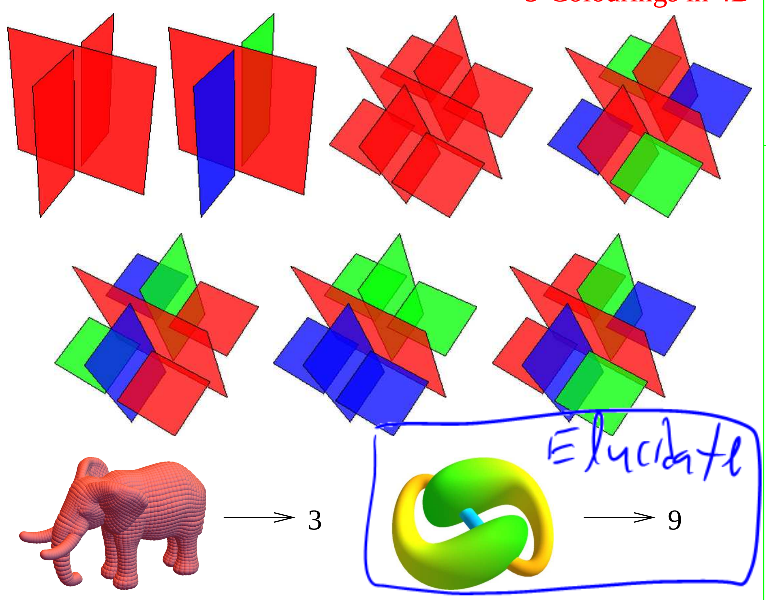
Meike Akveld and Andrew Jobbings, *Knots Unravelled, from Strings to Mathematics*, Arbelos 2011.

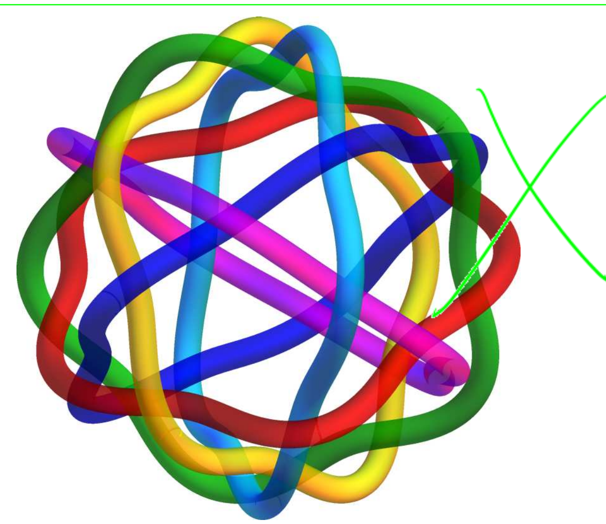
J. Scott Carter and Masahico Saito, *Knotted Surfaces and Their Diagrams*, American Mathematical Society, 1997.

Peter Cromwell, *Knots and Links*, Cambridge University Press, 2004.

W.B. Raymond Lickorish, *An Introduction to Knot Theory*, Springer 1997.

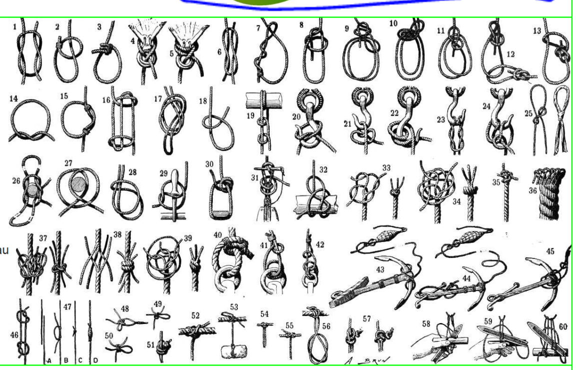
3-Colourings in 4D





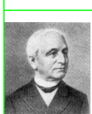
Banks like knots. Clean which knot appears twice?

Some knots for the practically-minded



Le Larousse pour tous : nouveau dictionnaire encyclopédique
by Larousse, Pierre, 1817-1875, Augé, Claude, 1854-1924
Published 1909

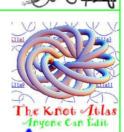
ω/LL



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org



* add some TI info.