Thomas Willwacher on Graph Cohomology

Friday, August 28, 2015 3:20 AM

First part j/w/ Fresse, Turchin

Want: $\mathrm{Emb}(M^m, N^n)$

Goodwillie-Weiss calculus: if $n-m\geq 3$

then

$$\mathrm{Emb}(M,n) \simeq \mathrm{Map}_{\mathrm{Mod}-E_m^{fr}}(\mathrm{Conf}(M), \mathrm{Conf}(N))$$

This is all wrong!! Why is the beautiful picture behind this never shown? We should not blindly follow our priests.

Goal: Express r.h.t. in terms of graph complexes.

Today: Base case: $M=\mathbb{R}^m$, $N=\mathbb{R}^n$

$$\overline{\mathrm{Emb}_\partial}(\mathbb{R}^m, \mathbb{R}^n)$$

∂: fixed at ends "long"

$\overline{}$: Embeddings along w/ a path to the unknot in immersions

$$\| $$

$$\mathrm{Map}_{E_m\text{-bimod}}(\mathrm{Conf}(\mathbb{R}^m), \mathrm{conf}(\mathbb{R}^n))$$

$\|$ Dwyer-Hess-Bellwald

$$\Omega^{m+1}\mathrm{Map}_{op}(E_m, E_n)$$

We settle for $\mathbb{Q}$:

$\mathrm{Map}_{op}(E_m, E_n^{\mathbb{Q}})$

Alg. setup: a model for a space is a dgca, contravariantly.

Model for top operad:

dg Hopf co-operad

$\equiv$ Cooperad in dgca's.

Technical point: "Λ-structure replacing nullary ops".

Define $\mathrm{Map}_{op}(E_m, E_n^{\mathbb{Q}})$:

— Pick rational models (dg Hopf cooperads) for E_m, E_n

M_m M_n

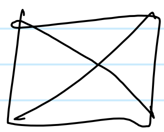
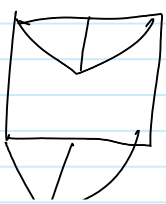
⋮

25:00

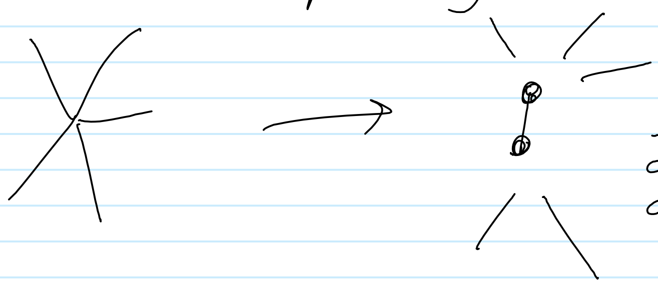
35:00

Graph complexes: GC_n

Series of isomorphism classes of graphs

 $+ \frac{1}{2}$  $+ \ldots$

differential: splitting vertices:



$\text{degree}(\Gamma) = (v-1)(-n) + e(n-1)$

so

$\deg(\delta) = -1$

really GC^2 meaning vertices are at least bivalent.

- Hairy graph complexes: $HGC_{m,n}$

same, w/ univalent ends.

the degree of hair ends is $(-m)$

$HGC_{m,n}$ is a dg Lie algebra

(bracket in video, 40:00–50:00)

Results (w/ Fresse, Turchin)

Thm For $n \geq m \geq 2$,

$$\mathrm{Map}(E_m, E_n^{\mathbb{Q}}) \simeq MC_\bullet(HGC_{m,n})$$

For $n \geq 2$, $m = 1$:

$$\mathrm{Map}(E_1, E_n^{\mathbb{Q}}) \simeq MC_\bullet(HGC'_{m,n})$$

Equipped w/ shortcut L_∞-structure

57:00