Sergei Merkulov on An Explicit Formula for Deformation Quantization of Lie BiAlgebras

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Part I

Quantization of Poisson Structures

Poisson str. on $\mathbb{R}^d$ $[\pi,\pi]_S=0$ $\xrightarrow{\text{Kontsevich}}$ $*_A$ product in $C^\infty(\mathbb{R}^d)$ $f *_A g = fg + \hbar\pi(f,g)$

Later w/ different propagator: [ARTW]: Alekseev Rossi ...

Then Tamarkin's existence

Today: a new formula, depending on an intermediate object:

Poisson structures $\xleftrightarrow{\text{Associators}}$ Quantizable Poisson structures $\longleftrightarrow$ $*_\hbar$

Part II A new explicit formula for defQ of Lie BiAlgebras.

History: Etingof-Kazhdan, Tamarkin, Severa, claimed explicit formula by Shoikhet, now a new formula:

$\boxed{\text{ordinary Lie BiAlg}} \xleftrightarrow{\text{ASS}} \boxed{\text{Quantizable Lie BiAlg}} \xleftrightarrow{\text{"cheap"}} \boxed{*_\hbar \ \Delta_\hbar}$

Part I

(in $\mathbb{R}^d$)

Quantizable Poisson structure

$$\pi = \sum \pi^{ij}(x)\frac{\partial}{\partial x^i}\wedge\frac{\partial}{\partial x^j} \qquad \psi_i = \frac{\partial}{\partial x_i} \qquad |\psi_i| = 1$$

"degree"

$$T_{poly}(\mathbb{R}^d) = C^\infty(x)[\psi_1 \dots \psi_d]$$

$$[f,g] = \sum \frac{\partial f}{\partial x^i}\frac{\partial g}{\partial \psi_i} - \text{swap}$$

Ordinary poisson str: $[\pi,\pi]_0 = 0$

quantizable Poisson str: Formal power series

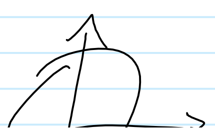
$$\pi^Q = \sum \hbar^k \pi^k(x,\psi) \quad \text{s.t.}$$

$$0 = [\pi^Q,\pi^Q] + \hbar^2[\pi^Q,\pi^Q,\pi^Q,\pi^Q]_4 + \hbar^4[\quad]_6 + \dots$$

"Kontsevich–Shoikhet Lie_∞ str."

⋮

Explicit formula for $[\quad]_{2k}$



$\text{Value} = \frac{1}{2\pi} d\,\text{Arg}\,z$