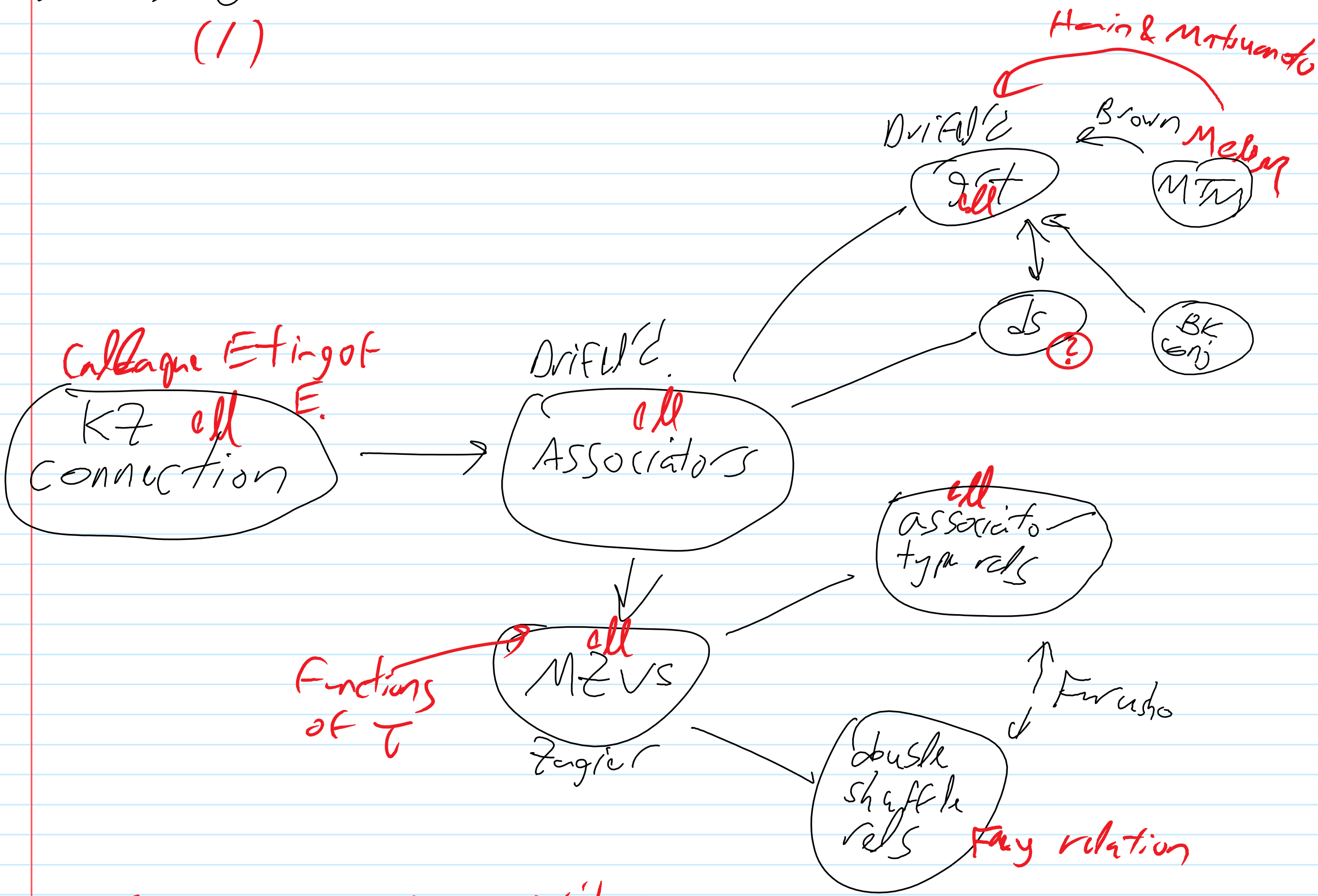
Enriquez on Elliptic Associators

Sunday, August 23, 2015 10:55 AM

Genus 0:

(1)



more names on video....

Genus 0

① KZ connection over $Cf_n(\mathbb{C}) = \{(z_i) : z_i \neq z_j\}$

w/ values in $\exp(\mathfrak{t}_n)$

$$\mathfrak{t}_n = \langle t_{ij} : 1 \leq i \neq j \leq n \rangle / \text{rels}$$

$t_{ij} = t_{ji}$, 6C, 4T

s.t. $d - \sum_{i \neq j} t_{ij}\, d\log(z_i - z_j)$ is flat.

S_n acts, get

$$\mathrm{mon}_n: B_n(\mathbb{C}) = \pi_1\left(\frac{Cf_n(\mathbb{C})}{S_n}\right) \longrightarrow \exp(\hat{t}_n) \rtimes S_n$$

w/ assymptotic basepoints

2. Associators

$$\mathrm{mon} = \mathrm{alg}(2\pi i, \phi_{KZ})$$

$\phi_{KZ} \in \mathbb{C}\langle\langle A, B\rangle\rangle$ is the holonomy of the

diff eq $\quad G'(z) = \left(\frac{A}{z} + \frac{B}{1-z}\right) G(z)$

⋮

example $\quad B_3(\mathbb{C}) \longrightarrow \exp \hat{t}_3 \rtimes S_3$

w/ $\quad \sigma_1 \longmapsto e^{i\pi t_{12}} \cdot (12)$

$\quad \sigma_2 \longmapsto \phi(t_{12}, t_{23})^{-1} e^{i\pi t_{23}} (23)\, \phi(t_{12}, t_{23})$.

Associator: A series in two variables which satisfies the same conclusion

Really,

$$\left\{(\mu, \phi) : \begin{array}{l} \langle\text{pentagon}\rangle \text{ in } \exp(\hat{t}_4) \\ \left.\begin{array}{l}\langle\text{hexagon}\rangle \\ \langle\text{duality}\rangle\end{array}\right\} \text{ in } \exp(\hat{t}_3) \end{array}\right\} = \underline{M}(\mathbb{C})$$

Likewise ... $\underline{M}(K)$ $\quad K$: a $\mathbb{Q}$-ring.

so we have a scheme of associators / $\mathbb{Q}$

3. MZV's:

$\zeta(\ldots) = \sum \frac{1}{\ldots}$

$$\zeta(s_1 \dots s_k) = \sum_{n_1 < n_2 < \dots < n_k} \frac{1}{n_1^{s_1} \cdots n_k^{s_1}}$$

((defined for $s_1 \geq 1 \dots s_{k-1} \geq 1$ $s_k \geq 2$

$\int_0^1$ some iterated integral.

So Φ_{KZ} can be expressed using MZV's.

So relations between associators give relations between MZV's.

There are also double-shuffle (ds) relations.

Furusho: Associator relations imply the ds rels.

4. Symmetries $M(\underline{k})$ has a categorical interpretations.

monoidal, braided monoidal, infinitesimal braided . . .

$$\{IBMC\} \xrightarrow{assoc} \{BMC\}$$

"Double shuffles are also a (bit$_0$) torsor"!

Fact: $\operatorname{spec} \mathbb{Q}[\zeta^{\text{formal}}(n_1 \ldots n_s)] / \text{double shuffle rels}$

$\quad\quad ||$

DMR a $\mathbb{Q}$-scheme is a torsor over the ds Lie algebra.

$ds := \{\psi \in L(a,b) : \psi$ is also primitive when written in terms of $A^k B,\ k \geq 0\}$

Hour 2: Elliptic KZB connection

E = elliptic curve $= \mathbb{C} / \mathbb{Z} + \tau\mathbb{Z}$ $\quad \operatorname{im} \tau > 0$

$Cf_n(E) = E^n \setminus$ diagonals $= \ldots$

$t_{1,n}$: the genus 1 analogue of t_1:

$$\left\langle \begin{matrix} x_i, y_i \\ t_{ij} \end{matrix} \right\rangle \Big/ \begin{matrix} \sum x_i = \sum y_i = 0 \\ [x_i, y_j] = t_{ij} = [x_j, y_i] \\ [x_i, t_{jk}] = [y_i, t_{jk}] = 0 \end{matrix}$$

$\cup$

t_1

I couldn't do this, but I also wouldn't. Given that I'm stupider, does it mean that my taste is just less developed?

BBCs, iBMCs, & Elliptic structures over those should have appeared in this lecture.