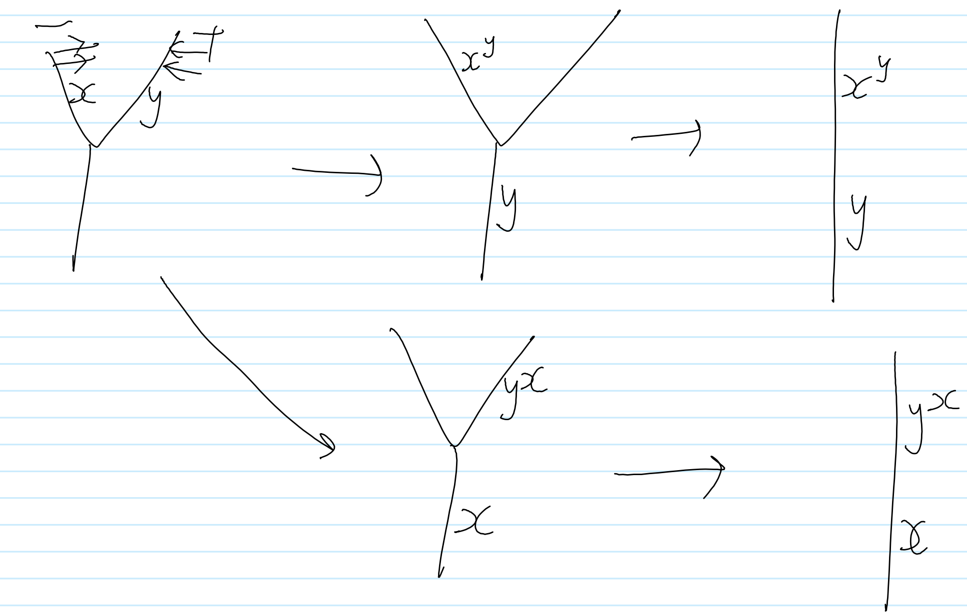
EK Multiplication

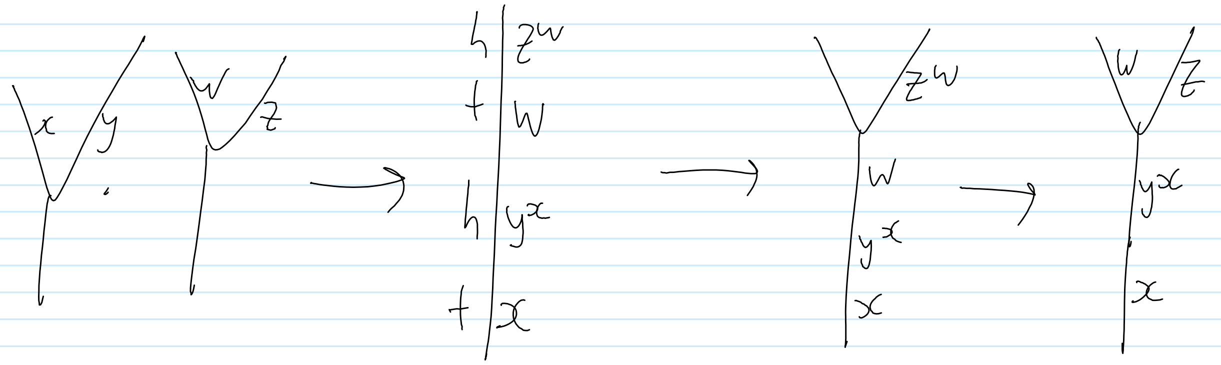
Saturday, August 15, 2015 11:49 AM

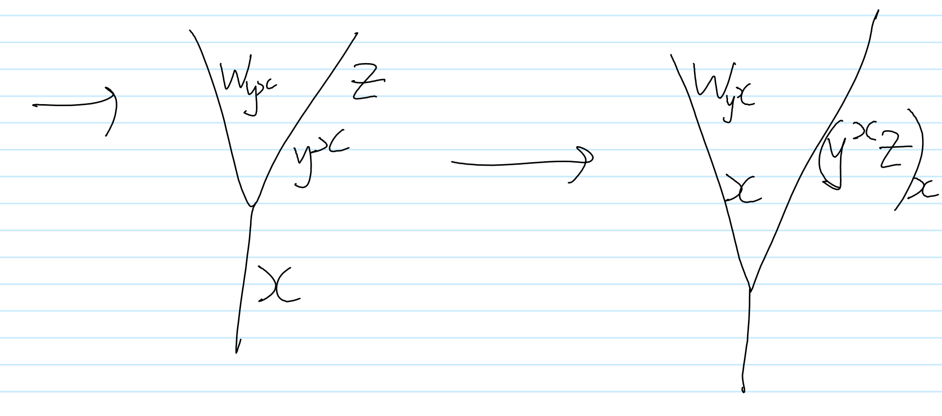
From Etingof-Kazhdan, I:

Lemma 2.1. *The assignment* $1 \to 1_+ \otimes 1_-$ *extends to an isomorphism of* $\mathfrak{g}$*-modules* $\phi : U(\mathfrak{g}) \to M_+ \otimes M_-$.

Proof. Since $M_\pm$ has been identified with $U(\mathfrak{g}_\mp)$, we can regard the map ϕ as a linear map $U(\mathfrak{g}) \to U(\mathfrak{g}_-) \otimes U(\mathfrak{g}_+)$. It is clear that this map preserves the standard filtration, so it defines a map of the associated graded objects: $S\mathfrak{g} \to S\mathfrak{g}_- \otimes S\mathfrak{g}_+$. This map is the isomorphism induced by the isomorphism $\mathfrak{g} \to \mathfrak{g}_- \oplus \mathfrak{g}_+$. Therefore, ϕ is an isomorphism. $\square$







Classic s.d.:
$(ab)(cd) = ac\, b^c d$
$= abcd$

quasi-direct products?

$$(a,b)(c,d) := \left(a\cdot c_{b^a}, (b^a d)_a\right)$$

is there a group theory analog, like bicrossed products?

junk.

$$\big((a\cdot b)\cdot(c\cdot d)\big)(e,f) = \left(a\cdot c_{b^a}, b d_a\right)(e,f)$$

$$= \left(a c_{b^a} e_{(bd_a)^{ac_{b^a}}}, b d_a f_{ac_{b^a}}\right)$$

$$(c,d)(e,f) = \left(c\cdot e_{d^c}, d f_c\right)$$

$$(a,b)\left[(c,d)(e,f)\right] = (a,b)\left(c e_{d^c}, d f_c\right) =$$

$$= \left(a(c e_{d^c})_{b^a}, b(d f_c)_a\right)$$

$$b d_a f_{ac_{b^a}} \overset{?}{=} b(d f_c)_a \qquad \text{need } (f_c)_a =$$