

VS = True;

Jacobi@@{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]}

{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]} → c[-f g h b₁ b₂ (ε₆ + ε₇), 1] +
c[f g h b₁ b₂ (ε₆ + ε₇), 2] + δa[-f g h b₁ (ε₆ + ε₇), 2, 2] + δa[f g h b₂ (ε₆ + ε₇), 1, 1]

Jacobi@@{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]} /. (erule = {ε₇ → -ε₆)

{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]} → 0

Jacobi@@{a[f, 1, 2], a[g, 1, 3], a[h, 2, 3]}

Tails commute on δaa[f g h, 1, 3, 1, 2]

Commute head/tail on δaa[f g h, 2, 3, 1, 2]

Commute heads on δaa[-f g h, 2, 3, 1, 3]

Commute heads on δaa[f g h, 2, 3, 1, 3]

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]} /. erule

1322 swinging on δaa[-2, 1, 3, 2, 2]

Commute head/tail on δaa[-2, 2, 3, 1, 2]

Commute heads on δaa[2, 2, 3, 1, 3]

Tails commute on δaa[-1, 1, 3, 1, 2]

1321 swinging on δaa[-1, 1, 3, 2, 1]

2113 swinging on δaa[1, 2, 1, 1, 3]

{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]} →

c[b₁ b₂ (-ε₁ + ε₆), 3] + δa[b₁ (-1 + ε₁ - 2 ε₂), 2, 3] + δa[b₂ (1 + 2 ε₂ - ε₆), 1, 3]

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]} /. (erule = erule ∪ {ε₆ → ε₁})

1322 swinging on δaa[-2, 1, 3, 2, 2]

Commute head/tail on δaa[-2, 2, 3, 1, 2]

Commute heads on δaa[2, 2, 3, 1, 3]

Tails commute on δaa[-1, 1, 3, 1, 2]

1321 swinging on δaa[-1, 1, 3, 2, 1]

2113 swinging on δaa[1, 2, 1, 1, 3]

{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]} →

δa[b₁ (-1 + ε₁ - 2 ε₂), 2, 3] + δa[b₂ (1 - ε₁ + 2 ε₂), 1, 3]

Solve[(-1 + ε₁ - 2 ε₂) == 0]

{ {ε₂ → - $\frac{1}{2}$ + $\frac{\epsilon_1}{2}$ } }

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]} /. (erule = erule ∪ {ε₂ → - $\frac{1}{2}$ + $\frac{\epsilon_1}{2}$ }) // S

1322 swinging on $\delta_{aa}[-2, 1, 3, 2, 2]$
 Commute head/tail on $\delta_{aa}[-2, 2, 3, 1, 2]$
 Commute heads on $\delta_{aa}[2, 2, 3, 1, 3]$
 Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$
 1321 swinging on $\delta_{aa}[-1, 1, 3, 2, 1]$
 2113 swinging on $\delta_{aa}[1, 2, 1, 1, 3]$
 $\{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]\} \rightarrow 0$

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]} /. erules // S

1332 swinging on $\delta_{aa}[-2, 1, 3, 3, 2]$
 Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$
 3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$
 1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$
 $\{a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]\} \rightarrow$
 $c[2 b_1 b_3 (1 + \epsilon_1), 2] + \delta a[-b_1 (1 + \epsilon_1), 3, 2] + \delta a[-b_3 (1 + \epsilon_1), 1, 2]$

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]} //. (erules = erules \cup { $\epsilon_1 \rightarrow -1$ }) // S

1332 swinging on $\delta_{aa}[-2, 1, 3, 3, 2]$
 Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$
 3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$
 1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$
 $\{a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]\} \rightarrow 0$

erules

$\{\epsilon_1 \rightarrow -1, \epsilon_2 \rightarrow -\frac{1}{2} + \frac{\epsilon_1}{2}, \epsilon_6 \rightarrow \epsilon_1, \epsilon_7 \rightarrow -\epsilon_6\}$

{ $\epsilon_1 = -1, \epsilon_2 = -1, \epsilon_6 = -1, \epsilon_7 = 1$ };

{x1, x2, x3} = {a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]}

{a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]}

S[{B[x1, B[x2, x3]] , B[x2, B[x3, x1]] , B[x3, B[x1, x2]]}]

1332 swinging on $\delta_{aa}[-2, 1, 3, 3, 2]$

1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$

Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$

3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$

$$\begin{aligned} & \{ a [b_1^2, 3, 2] + a [-b_1 b_3, 1, 2] + c [3 b_1 b_3, 2] + ca [-2 b_1, 1, 3, 2] + \\ & \quad ca [-2 b_1, 2, 3, 3] + ca [b_1, 2, 3, 1] + ca [b_1, 3, 3, 2] + ca [-2 b_3, 3, 1, 2] + \\ & \quad ca [-b_3, 2, 1, 1] + ca [b_3, 1, 1, 2] + ca [-b_1 + b_3, 2, 1, 3] + \delta a [-b_1, 3, 2] + \\ & \quad \delta a [-2 b_3, 1, 2] + \delta_{aa} [1, 1, 1, 3, 2] + \delta_{aa} [1, 1, 2, 1, 3] + \delta_{aa} [2, 1, 2, 3, 3], \\ & a [-b_1^2, 3, 2] + a [b_1 b_3, 1, 2] + c [-2 b_1 b_3, 2] + ca [-b_1, 2, 3, 1] + ca [b_1, 1, 3, 2] + \\ & \quad ca [b_1, 2, 1, 3] + ca [b_1, 2, 3, 3] + ca [-b_3, 1, 1, 2] + ca [b_3, 2, 1, 1] + \\ & \quad ca [b_3, 3, 1, 2] + \delta a [2 b_3, 1, 2] + \delta_{aa} [-2, 1, 2, 3, 3] + \delta_{aa} [-1, 1, 2, 1, 3], \\ & c [-b_1 b_3, 2] + ca [-b_1, 3, 3, 2] + ca [b_1, 1, 3, 2] + ca [b_1, 2, 3, 3] + \\ & \quad ca [-b_3, 2, 1, 3] + ca [b_3, 3, 1, 2] + \delta a [b_1, 3, 2] + \delta_{aa} [-1, 1, 1, 3, 2] \} \end{aligned}$$

B[x1, B[x2, x3]] // S

1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$

$$\begin{aligned} & a [b_1^2, 3, 2] + a [-b_1 b_3, 1, 2] + c [3 b_1 b_3, 2] + ca [-2 b_1, 1, 3, 2] + \\ & \quad ca [-2 b_1, 2, 3, 3] + ca [b_1, 2, 3, 1] + ca [b_1, 3, 3, 2] + ca [-2 b_3, 3, 1, 2] + \\ & \quad ca [-b_3, 2, 1, 1] + ca [b_3, 1, 1, 2] + ca [-b_1 + b_3, 2, 1, 3] + \delta a [-b_1, 3, 2] + \\ & \quad \delta a [-2 b_3, 1, 2] + \delta_{aa} [1, 1, 1, 3, 2] + \delta_{aa} [1, 1, 2, 1, 3] + \delta_{aa} [2, 1, 2, 3, 3] \end{aligned}$$

Jacobi@@{a[f[b1, b2, b3, b4], 1, 2], a[g[b1, b2, b3, b4], 1, 3], a[h[b1, b2, b3, b4], 3, 1]}

Tails commute on

$$\begin{aligned} & \delta_{aa} [-g [b_1, b_2, b_3, b_4] h [b_1, b_2, b_3, b_4] b_3 (f^{(0,0,1,0)} [b_1, b_2, b_3, b_4] - f^{(1,0,0,0)} [b_1, b_2, b_3, b_4]) - \\ & \quad f [b_1, b_2, b_3, b_4] g [b_1, b_2, b_3, b_4] b_3 \\ & \quad (-h^{(0,0,1,0)} [b_1, b_2, b_3, b_4] + h^{(1,0,0,0)} [b_1, b_2, b_3, b_4]), 1, 3, 1, 2] \end{aligned}$$

1332 swinging on $\delta_{aa}[-f[b_1, b_2, b_3, b_4] g[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] + g[b_1, b_2, b_3, b_4]$

$$\begin{aligned} & (h [b_1, b_2, b_3, b_4] b_1 (f^{(0,0,1,0)} [b_1, b_2, b_3, b_4] - f^{(1,0,0,0)} [b_1, b_2, b_3, b_4]) - f [b_1, b_2, b_3, b_4] \\ & \quad (h [b_1, b_2, b_3, b_4] + b_1 (-h^{(0,0,1,0)} [b_1, b_2, b_3, b_4] + h^{(1,0,0,0)} [b_1, b_2, b_3, b_4]))) , 1, 3, 3, 2] \end{aligned}$$

Tails commute on $\delta_{aa}[2 g[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4]$

$$b_3 (f^{(0,0,1,0)} [b_1, b_2, b_3, b_4] - f^{(1,0,0,0)} [b_1, b_2, b_3, b_4]), 1, 3, 1, 2]$$

3112 swinging on

$$\begin{aligned} & \delta_{aa} [-g [b_1, b_2, b_3, b_4] h [b_1, b_2, b_3, b_4] b_1 (f^{(0,0,1,0)} [b_1, b_2, b_3, b_4] - f^{(1,0,0,0)} [b_1, b_2, b_3, b_4]) - \\ & \quad g [b_1, b_2, b_3, b_4] h [b_1, b_2, b_3, b_4] \\ & \quad (f [b_1, b_2, b_3, b_4] + b_1 (-f^{(0,0,1,0)} [b_1, b_2, b_3, b_4] + f^{(1,0,0,0)} [b_1, b_2, b_3, b_4])) , 3, 1, 1, 2] \end{aligned}$$

Commute head/tail on

$$\begin{aligned} & \delta_{aa} [-f [b_1, b_2, b_3, b_4] h [b_1, b_2, b_3, b_4] b_1 (-g^{(0,0,1,0)} [b_1, b_2, b_3, b_4] + g^{(0,1,0,0)} [b_1, b_2, b_3, b_4]) + \\ & \quad f [b_1, b_2, b_3, b_4] h [b_1, b_2, b_3, b_4] b_1 \\ & \quad (-g^{(0,0,1,0)} [b_1, b_2, b_3, b_4] + g^{(1,0,0,0)} [b_1, b_2, b_3, b_4]), 3, 2, 1, 3] \end{aligned}$$

Commute heads on

$$\begin{aligned} & \delta_{aa}[f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 (-g^{(0,0,1,0)}[b_1, b_2, b_3, b_4] + g^{(0,1,0,0)}[b_1, b_2, b_3, b_4]) - \\ & f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 \\ & (-g^{(0,0,1,0)}[b_1, b_2, b_3, b_4] + g^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 3, 2, 1, 2] \end{aligned}$$

1332 swinging on

$$\begin{aligned} & \delta_{aa}[-f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 (-g^{(0,0,1,0)}[b_1, b_2, b_3, b_4] + g^{(0,1,0,0)}[b_1, b_2, b_3, b_4]) + \\ & f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 \\ & (-g^{(0,0,1,0)}[b_1, b_2, b_3, b_4] + g^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 1, 3, 3, 2] \end{aligned}$$

1332 swinging on

$$\begin{aligned} & \delta_{aa}[-g[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 (f^{(0,0,1,0)}[b_1, b_2, b_3, b_4] - f^{(1,0,0,0)}[b_1, b_2, b_3, b_4]) + \\ & f[b_1, b_2, b_3, b_4] g[b_1, b_2, b_3, b_4] b_1 \\ & (-h^{(0,0,1,0)}[b_1, b_2, b_3, b_4] + h^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 1, 3, 3, 2] \end{aligned}$$

3112 swinging on

$$\begin{aligned} & \delta_{aa}[g[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 (f^{(0,0,1,0)}[b_1, b_2, b_3, b_4] - f^{(1,0,0,0)}[b_1, b_2, b_3, b_4]) + \\ & g[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 \\ & (-f^{(0,0,1,0)}[b_1, b_2, b_3, b_4] + f^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 3, 1, 1, 2] \end{aligned}$$

1231 swinging on

$$\begin{aligned} & \delta_{aa}[-f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4] b_1 (-g^{(0,1,0,0)}[b_1, b_2, b_3, b_4] + g^{(1,0,0,0)}[b_1, b_2, b_3, b_4]) + \\ & f[b_1, b_2, b_3, b_4] g[b_1, b_2, b_3, b_4] b_1 \\ & (h^{(0,1,0,0)}[b_1, b_2, b_3, b_4] - h^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 1, 2, 3, 1] \end{aligned}$$

Commute head/tail on $\delta_{aa}[-f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4]$

$$b_1 (-g^{(0,1,0,0)}[b_1, b_2, b_3, b_4] + g^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 3, 2, 1, 3]$$

Commute heads on $\delta_{aa}[f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4]$

$$b_1 (-g^{(0,1,0,0)}[b_1, b_2, b_3, b_4] + g^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 3, 2, 1, 2]$$

1332 swinging on $\delta_{aa}[-f[b_1, b_2, b_3, b_4] h[b_1, b_2, b_3, b_4]$

$$b_1 (-g^{(0,1,0,0)}[b_1, b_2, b_3, b_4] + g^{(1,0,0,0)}[b_1, b_2, b_3, b_4]), 1, 3, 3, 2]$$

Tails commute on $\delta_{aa}[g[b_1, b_2, b_3, b_4]$

$$\begin{aligned} & (h[b_1, b_2, b_3, b_4] b_3 (-f^{(0,0,1,0)}[b_1, b_2, b_3, b_4] + f^{(1,0,0,0)}[b_1, b_2, b_3, b_4]) - f[b_1, b_2, b_3, b_4] \\ & (h[b_1, b_2, b_3, b_4] + b_3 (h^{(0,0,1,0)}[b_1, b_2, b_3, b_4] - h^{(1,0,0,0)}[b_1, b_2, b_3, b_4]))) , 1, 3, 1, 2] \end{aligned}$$

1231 swinging on $\delta_{aa}[f[b_1, b_2, b_3, b_4]$

$$\begin{aligned} & (h[b_1, b_2, b_3, b_4] b_1 (-g^{(0,1,0,0)}[b_1, b_2, b_3, b_4] + g^{(1,0,0,0)}[b_1, b_2, b_3, b_4]) + g[b_1, b_2, b_3, b_4] \\ & (h[b_1, b_2, b_3, b_4] + b_1 (-h^{(0,1,0,0)}[b_1, b_2, b_3, b_4] + h^{(1,0,0,0)}[b_1, b_2, b_3, b_4]))) , 1, 2, 3, 1] \end{aligned}$$

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], ca[1, 1, 3, 1]}

0

Jacobi@@{a[f[b1, b2, b3], 1, 2], a[g[b1, b2, b3], 1, 3], $\delta_{aa}[h[b1, b2, b3], 3, 1, 3, 1]}$ }

0

```
Jacobi @@ {a[1, 1, 2], a[1, 2, 1], a[1, 3, 1]}
```

```
{a[1, 1, 2], a[1, 2, 1], a[1, 3, 1]} →
```

```
c[-2 b2 b3, 1] + ca[-2 b2, 1, 3, 2] + ca[2 b2, 2, 3, 1] + ca[-2 b3, 2, 2, 1] +  
ca[2 b3, 1, 2, 2] + δa[2 b3, 2, 1] + δaa[-2, 2, 2, 3, 1] + δaa[2, 2, 1, 3, 2]
```

```
VS = False;
```

```
JacErrors = DeleteCases[
```

```
  bas1 = FormalBasis[3, f];
```

```
  bas2 = FormalBasis[3, g];
```

```
  bas3 = FormalBasis[3, h];
```

```
  Flatten[
```

```
    Table[Jacobi[bas1[[i]], bas2[[j]], bas3[[k]],
```

```
      {i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}]
```

```
  ],
```

```
  0]
```

```
{a[f[b1, b2, b3], 1, 2], a[g[b1, b2, b3], 2, 1], a[h[b1, b2, b3], 3, 1]} →  
c[-2 f[b1, b2, b3] g[b1, b2, b3] h[b1, b2, b3] b2 b3, 1] + ... 6 ... +  
δaa[2 f[b1, b2, b3] g[b1, b2, b3] h[b1, b2, b3], 2, 1, 3, 2], ... 160 ... ,  
{a[f[b1, b2, b3], 3, 1], δaa[g[b1, b2, b3], 1, 3, 2, 3], a[h[b1, b2, b3], 3, 2]} →  
... 1 ... }
```

large output

show less

show more

show all

set size limit...

```
(* was 186 for FormalBasis[3], 2156 for FormalBasis[4] *)
```

```
Length@JacErrors
```

```
162
```