

VS = True;

Jacobi@@{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]}

0

Jacobi@@{a[f, 1, 2], a[g, 1, 3], a[h, 2, 3]}

Tails commute on $\delta_{aa}[fgh, 1, 3, 1, 2]$

Commute head/tail on $\delta_{aa}[fgh, 2, 3, 1, 2]$

Commute heads on $\delta_{aa}[-fgh, 2, 3, 1, 3]$

Commute heads on $\delta_{aa}[fgh, 2, 3, 1, 3]$

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]}

1322 swinging on $\delta_{aa}[-2, 1, 3, 2, 2]$

Commute head/tail on $\delta_{aa}[-2, 2, 3, 1, 2]$

Commute heads on $\delta_{aa}[2, 2, 3, 1, 3]$

Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$

1321 swinging on $\delta_{aa}[-1, 1, 3, 2, 1]$

2113 swinging on $\delta_{aa}[1, 2, 1, 1, 3]$

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]} // S

1332 swinging on $\delta_{aa}[-2, 1, 3, 3, 2]$

Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$

3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$

1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], ca[1, 1, 3, 1]}

1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$

1332 swinging on $\delta_{aa}[b_1, 1, 3, 3, 2]$

Tails commute on $\delta_{aa}[-b_3, 1, 3, 1, 2]$

1231 swinging on $\delta_{aa}[-b_1, 1, 2, 3, 1]$

1211 sliding on $\delta_{aa}[b_3, 1, 2, 1, 1]$

0

Jacobi@@{a[f[b₁, b₂, b₃], 1, 2], a[g[b₁, b₂, b₃], 1, 3], $\delta_{aa}[h[b_1, b_2, b_3], 3, 1, 3, 1]}$

Tails commute on $\delta_{aa}[-f[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1, 3, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[f[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3, 1, 2, 3, 1]$
 3112 swinging on $\delta_{aa}[f[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3, 3, 1, 1, 2]$
 1332 swinging on $\delta_{aa}[4 f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 3, 3, 2]$
 2221 sliding on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1^2, 3, 3, 3, 2]$
 Tails commute on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3^2, 1, 3, 1, 2]$
 Tails commute on $\delta_{aa}[2 f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1^2, 3, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[-2 f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 2, 3, 1]$
 1332 swinging on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 3, 3, 2]$
 Locality on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 2, 1, 1]$
 Commute head/tail on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 2, 1, 3]$
 Commute heads on $\delta_{aa}[f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 2, 1, 2]$
 1332 swinging on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 3, 3, 2]$
 Locality on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 3, 1, 2]$
 1211 sliding on $\delta_{aa}[f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3^2, 1, 2, 1, 1]$
 0

Jacobi@@{a[1, 1, 2], a[1, 2, 1], a[1, 3, 1]}

Locality on $\delta_{aa}[-2, 3, 1, 2, 2]$
 2231 swinging on $\delta_{aa}[-2, 2, 2, 3, 1]$
 3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$
 Commute heads on $\delta_{aa}[-1, 3, 1, 2, 1]$
 1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$
 0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], $\delta_{aa}[1, 2, 1, 3, 1]$ }

2231 swinging on $\delta_{aa}[-b_1, 2, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[b_2, 1, 2, 3, 1]$
 Backie swinging on $\delta_{aa}[b_3, 2, 1, 1, 2]$
 Locality on $\delta_{aa}[b_3, 2, 2, 1, 1]$
 Backie swinging on $\delta_{aa}[-b_1^2, 2, 3, 3, 2]$
 1332 swinging on $\delta_{aa}[b_1 b_2, 1, 3, 3, 2]$
 Backie swinging on $\delta_{aa}[b_1 b_3, 2, 1, 1, 2]$
 Locality on $\delta_{aa}[b_1 b_3, 2, 2, 1, 1]$
 Tails commute on $\delta_{aa}[-b_2 b_3, 1, 3, 1, 2]$
 2231 swinging on $\delta_{aa}[b_1^2, 2, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[-b_1 b_2, 1, 2, 3, 1]$
 Locality on $\delta_{aa}[-b_1 b_3, 2, 2, 1, 1]$
 Locality on $\delta_{aa}[-b_1 b_3, 2, 2, 1, 3]$
 1322 swinging on $\delta_{aa}[-b_1 b_3, 1, 3, 2, 2]$
 Commute head/tail on $\delta_{aa}[-b_1 b_3, 2, 3, 1, 2]$
 Commute heads on $\delta_{aa}[b_1 b_3, 2, 3, 1, 3]$
 Commute head/tail on $\delta_{aa}[-b_1 b_3, 2, 3, 1, 2]$
 Commute heads on $\delta_{aa}[b_1 b_3, 2, 3, 1, 3]$
 1322 swinging on $\delta_{aa}[2 b_1 b_3, 1, 3, 2, 2]$
 Commute head/tail on $\delta_{aa}[2 b_1 b_3, 2, 3, 1, 2]$
 Commute heads on $\delta_{aa}[-2 b_1 b_3, 2, 3, 1, 3]$
 1211 sliding on $\delta_{aa}[b_2 b_3, 1, 2, 1, 1]$
 0

Jacobi @@ {a[1, 1, 2], a[1, 2, 1], a[1, 3, 2]}

1231 swinging on $\delta_{aa}[-2, 1, 2, 3, 1]$
 Locality on $\delta_{aa}[2, 3, 2, 1, 1]$
 Commute heads on $\delta_{aa}[1, 3, 2, 1, 2]$
 $\{a[1, 1, 2], a[1, 2, 1], a[1, 3, 2]\} \rightarrow$
 $\delta a[b_2, 3, 1] + \delta a[-b_3, 2, 1] + \delta_{aa}[-1, 2, 1, 3, 2] + \delta_{aa}[1, 3, 2, 2, 1]$

```

VS = False;
JacErrors = DeleteCases[
  bas1 = FormalBasis[3, f];
  bas2 = FormalBasis[3, g];
  bas3 = FormalBasis[3, h];
  Flatten[
    Table[Jacobi[bas1[[i]], bas2[[j]], bas3[[k]],
      {i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}]
  ],
0]

```

$\{ \{ a[f[b_1, b_2, b_3], 1, 2], a[g[b_1, b_2, b_3], 2, 1], a[h[b_1, b_2, b_3], 3, 2] \} \rightarrow$
 $\delta a[f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_2, 3, 1] +$
 $\delta a[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3, 2, 1] + \delta a a[\dots 1 \dots] +$
 $\delta a a[f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3], 3, 2, 2, 1],$
 $\dots 191 \dots, \{ a[f[b_1, b_2, b_3], 3, 1], \dots 1 \dots, a[\dots 1 \dots] \} \rightarrow \dots 1 \dots \}$

large output | show less | show more | show all | set size limit...

(* was 180 for FormalBasis[3], 2156 for FormalBasis[4]*)

Length@JacErrors

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