

VS = True;

Jacobi@@{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]}

{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]} →
 $c[f g h b_1 b_2 (\epsilon_5 \epsilon_6 + \epsilon_7), 2] + c[-f g h b_1 b_2 (\epsilon_6 + \epsilon_5 \epsilon_7), 1] +$
 $\delta a[-f g h b_1 (-1 + \epsilon_5) \epsilon_6, 2, 1] + \delta a[f g h b_2 (-1 + \epsilon_5) \epsilon_7, 1, 2] +$
 $\delta a[-f g h b_1 \epsilon_5 (\epsilon_6 + \epsilon_7), 2, 2] + \delta a[f g h b_2 \epsilon_5 (\epsilon_6 + \epsilon_7), 1, 1]$

Jacobi@@{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]} /. (erules = { $\epsilon_5 \rightarrow 1$, $\epsilon_7 \rightarrow -\epsilon_6$ })

{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]} → 0

Jacobi@@{a[f, 1, 2], a[g, 1, 3], a[h, 2, 3]}

Tails commute on $\delta a a[f g h, 1, 3, 1, 2]$
 Commute head/tail on $\delta a a[f g h, 2, 3, 1, 2]$
 Commute heads on $\delta a a[-f g h, 2, 3, 1, 3]$
 Commute heads on $\delta a a[f g h, 2, 3, 1, 3]$
 0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]} /. erules

1322 swinging on $\delta a a[-\epsilon_4 - \epsilon_5, 1, 3, 2, 2]$
 Commute head/tail on $\delta a a[-\epsilon_4 - \epsilon_5, 2, 3, 1, 2]$
 Commute heads on $\delta a a[\epsilon_4 + \epsilon_5, 2, 3, 1, 3]$
 2113 swinging on $\delta a a[1, 2, 1, 1, 3]$
 Tails commute on $\delta a a[-\epsilon_4, 1, 3, 1, 2]$
 1321 swinging on $\delta a a[-\epsilon_5, 1, 3, 2, 1]$
 $\{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]\} \rightarrow c[b_1 b_2 (-\epsilon_1 + \epsilon_6), 3] +$
 $ca[(b_1 + b_2) (-1 + \epsilon_4), 3, 1, 2] + \delta a[b_1 (\epsilon_1 + \epsilon_4 - (1 + \epsilon_2) (1 + \epsilon_4)), 2, 3] +$
 $\delta a[b_2 (1 + \epsilon_2 (1 + \epsilon_4) - \epsilon_6), 1, 3] + \delta a a[1 - \epsilon_4, 1, 2, 1, 3] + \delta a a[1 - \epsilon_4, 1, 2, 2, 3]$

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]} /. (erules = erules \cup { $\epsilon_4 \rightarrow 1$, $\epsilon_6 \rightarrow \epsilon_1$ })

1322 swinging on $\delta a a[-\epsilon_4 - \epsilon_5, 1, 3, 2, 2]$
 Commute head/tail on $\delta a a[-\epsilon_4 - \epsilon_5, 2, 3, 1, 2]$
 Commute heads on $\delta a a[\epsilon_4 + \epsilon_5, 2, 3, 1, 3]$
 2113 swinging on $\delta a a[1, 2, 1, 1, 3]$
 Tails commute on $\delta a a[-\epsilon_4, 1, 3, 1, 2]$
 1321 swinging on $\delta a a[-\epsilon_5, 1, 3, 2, 1]$
 $\{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]\} \rightarrow$
 $\delta a[b_2 (1 - \epsilon_1 + 2 \epsilon_2), 1, 3] + \delta a[b_1 (1 + \epsilon_1 - 2 (1 + \epsilon_2)), 2, 3]$

Solve $[(1 - \epsilon_1 + 2 \epsilon_2) == 0 \wedge (1 + \epsilon_1 - 2 (1 + \epsilon_2)) == 0]$

$\{\{\epsilon_2 \rightarrow -\frac{1}{2} + \frac{\epsilon_1}{2}\}\}$

Jacobi @@ $\{\mathbf{a}[1, 1, 2], \mathbf{a}[1, 1, 3], \mathbf{a}[1, 2, 1]\}$ /. $(\mathbf{erules} = \mathbf{erules} \cup \{\epsilon_2 \rightarrow -\frac{1}{2} + \frac{\epsilon_1}{2}\})$ // **S**

1322 swinging on $\delta_{aa}[-\epsilon_4 - \epsilon_5, 1, 3, 2, 2]$

Commute head/tail on $\delta_{aa}[-\epsilon_4 - \epsilon_5, 2, 3, 1, 2]$

Commute heads on $\delta_{aa}[\epsilon_4 + \epsilon_5, 2, 3, 1, 3]$

2113 swinging on $\delta_{aa}[1, 2, 1, 1, 3]$

Tails commute on $\delta_{aa}[-\epsilon_4, 1, 3, 1, 2]$

1321 swinging on $\delta_{aa}[-\epsilon_5, 1, 3, 2, 1]$

$\{\mathbf{a}[1, 1, 2], \mathbf{a}[1, 1, 3], \mathbf{a}[1, 2, 1]\} \rightarrow 0$

Jacobi @@ $\{\mathbf{a}[1, 1, 2], \mathbf{a}[1, 1, 3], \mathbf{a}[1, 3, 1]\}$ /. **erules** // **S**

1332 swinging on $\delta_{aa}[-2, 1, 3, 3, 2]$

Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$

3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$

1231 swinging on $\delta_{aa}[\epsilon_5, 1, 2, 3, 1]$

$\{\mathbf{a}[1, 1, 2], \mathbf{a}[1, 1, 3], \mathbf{a}[1, 3, 1]\} \rightarrow$

$c[2 b_1 b_3, 2] + \delta a[-b_1 (1 + \epsilon_1), 3, 2] + \delta a[-b_3 (1 + \epsilon_1), 1, 2]$

erules

$\{\epsilon_2 \rightarrow -\frac{1}{2} + \frac{\epsilon_1}{2}, \epsilon_4 \rightarrow 1, \epsilon_5 \rightarrow 1, \epsilon_6 \rightarrow \epsilon_1, \epsilon_7 \rightarrow -\epsilon_6\}$

$\{\epsilon_2 = -\frac{1}{2} + \frac{\epsilon_1}{2}, \epsilon_4 = 1, \epsilon_5 = 1, \epsilon_6 = \epsilon_1, \epsilon_7 = -\epsilon_6\};$

$\{\mathbf{x1}, \mathbf{x2}, \mathbf{x3}\} = \{\mathbf{a}[1, 1, 2], \mathbf{a}[1, 1, 3], \mathbf{a}[1, 3, 1]\}$

$\{\mathbf{a}[1, 1, 2], \mathbf{a}[1, 1, 3], \mathbf{a}[1, 3, 1]\}$

S $\{\{\mathbf{B}[\mathbf{x1}, \mathbf{B}[\mathbf{x2}, \mathbf{x3}]] , \mathbf{B}[\mathbf{x2}, \mathbf{B}[\mathbf{x3}, \mathbf{x1}]] , \mathbf{B}[\mathbf{x3}, \mathbf{B}[\mathbf{x1}, \mathbf{x2}]]\}\}$

```

1332 swinging on δaa[-2, 1, 3, 3, 2]
1231 swinging on δaa[1, 1, 2, 3, 1]
Tails commute on δaa[-1, 1, 3, 1, 2]
3112 swinging on δaa[-1, 3, 1, 1, 2]
{a[b1^2, 3, 2] + a[-b1 b3, 1, 2] + c[-b1 b3 (-2 + ε1), 2] + ca[-2 b1, 1, 3, 2] +
  ca[-2 b1, 2, 3, 3] + ca[b1, 2, 3, 1] + ca[b1, 3, 3, 2] + ca[-2 b3, 3, 1, 2] +
  ca[-b3, 2, 1, 1] + ca[b3, 1, 1, 2] + ca[-b1 + b3, 2, 1, 3] + δa[-b1, 3, 2] +
  δa[b3 (-1 + ε1), 1, 2] + δaa[1, 1, 1, 3, 2] + δaa[1, 1, 2, 1, 3] + δaa[2, 1, 2, 3, 3],
a[-b1^2, 3, 2] + a[b1 b3, 1, 2] + ca[-b1, 2, 3, 1] + ca[b1, 1, 3, 2] +
  ca[b1, 2, 1, 3] + ca[b1, 2, 3, 3] + ca[-b3, 1, 1, 2] + ca[b3, 2, 1, 1] +
  ca[b3, 3, 1, 2] + δa[-2 b3 ε1, 1, 2] + δaa[-2, 1, 2, 3, 3] + δaa[-1, 1, 2, 1, 3],
c[b1 b3 ε1, 2] + ca[-b1, 3, 3, 2] + ca[b1, 1, 3, 2] + ca[b1, 2, 3, 3] +
  ca[-b3, 2, 1, 3] + ca[b3, 3, 1, 2] + δa[-b1 ε1, 3, 2] + δaa[-1, 1, 1, 3, 2]}

```

B[x1, B[x2, x3]] // S

```

1231 swinging on δaa[1, 1, 2, 3, 1]
a[b1^2, 3, 2] + a[-b1 b3, 1, 2] + c[-b1 b3 (-2 + ε1), 2] + ca[-2 b1, 1, 3, 2] +
  ca[-2 b1, 2, 3, 3] + ca[b1, 2, 3, 1] + ca[b1, 3, 3, 2] + ca[-2 b3, 3, 1, 2] +
  ca[-b3, 2, 1, 1] + ca[b3, 1, 1, 2] + ca[-b1 + b3, 2, 1, 3] + δa[-b1, 3, 2] +
  δa[b3 (-1 + ε1), 1, 2] + δaa[1, 1, 1, 3, 2] + δaa[1, 1, 2, 1, 3] + δaa[2, 1, 2, 3, 3]

```

VS = False;

JacErrors = DeleteCases[

bas1 = FormalBasis[4, f];

bas2 = FormalBasis[4, g];

bas3 = FormalBasis[4, h];

Flatten[

Table[Jacobi[bas1[[i], bas2[[j], bas3[[k]],

{i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}]

],

0]

```

{a[f[b1, b2, b3, b4], 1, 2], a[g[b1, b2, b3, b4], 1, 3], a[h[b1, b2, b3, b4], 3, 1]} ->
  c[2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b1 b3, 2] +
  δa[-f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b1 (1 + ε1), 3, 2] +
  δa[-f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b3 (1 + ε1), 1, 2],
... 2154 ..., { ... 1 ... } -> { ... 1 ... }

```

large output

show less

show more

show all

set size limit...

(* was 1737 *) **Length@JacErrors**

2156

JE2 = DeleteCases[JacErrors /. { $\epsilon_6 \rightarrow 1$ }, _ $\rightarrow 0$]

```

{ {a[f[b1, b2, b3, b4], 1, 2], a[g[b1, b2, b3, b4], 1, 3], a[h[b1, b2, b3, b4], 3, 1]}  $\rightarrow$ 
ca[-2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b1, 2, 3, 3] + ... 8 ... +
 $\delta$ aa[2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4], 1, 2, 3, 3], ... 1735 ... ,
{a[f[b1, b2, b3, b4], 4, 2],  $\delta$ aa[... 1 ...], a[h[... 1 ...], 4, 3]}  $\rightarrow$  ... 1 ...

```

| | | | | |
|--------------|------------------|------------------|-----------------|--------------------------|
| large output | show less | show more | show all | set size limit... |
|--------------|------------------|------------------|-----------------|--------------------------|

Length@JE2

1737

JE2[[1]] // S

```

{a[f[b1, b2, b3, b4], 1, 2], a[g[b1, b2, b3, b4], 1, 3], a[h[b1, b2, b3, b4], 3, 1]}  $\rightarrow$ 
ca[-2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b1, 2, 3, 3] +
ca[2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b1, 3, 3, 2] +
ca[-2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b3, 3, 1, 2] +
ca[2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b3, 2, 1, 3] +
 $\delta$ a[-f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b1, 3, 2] +
 $\delta$ a[f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4] b3, 1, 2] +
 $\delta$ aa[-2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4], 1, 3, 3, 2] +
 $\delta$ aa[-f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4], 3, 1, 1, 2] +
 $\delta$ aa[f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4], 1, 2, 3, 1] +
 $\delta$ aa[2 f[b1, b2, b3, b4] g[b1, b2, b3, b4] h[b1, b2, b3, b4], 1, 2, 3, 3]

```

Sort[{i, j, jj, k}]

{i, j, jj, k}