Pensieve header: Demo for the package FreeLie`. More at http://www.math.toronto.edu/~drorbn/Talks/LesDiablerets-1508/

Note: The web root for my "Academic Pensieve" is http://drorbn.net/AcademicPensieve/. The packages FreeLie` and AwCalculus` are at http://drorbn.net/AcademicPensieve/Projects/WKO4/.

```
path = "C:/drorbn/AcademicPensieve/";
SetDirectory[path <> "2015-08/LesDiablerets-1508"];
Get[path <> "Projects/WKO4/FreeLie.m"];
Get[path <> "Projects/WKO4/AwCalculus.m"];
x = LW@"x"; y = LW@"y"; u = LW@"u";
$SeriesShowDegree = 6;
```

FreeLie` implements / extends

$\{*, +, **, \$SeriesShowDegree, \langle\rangle, \int, \equiv, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,$

AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW,

CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp,

Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,

Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,

$t, tb, TopBracketForm, tr, UndeterminedCoefficients, \alpha Map, \Gamma, \iota, \Lambda, \sigma, \hbar, \frown, \smile\}.$

FreeLie` is in the public domain. Dror Bar-Natan is committed

to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es,

hA, hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed

to support it within reason until July 15, 2022. This is version 150814.

BCH

```
BCH[x, y] (* Can raise degree to 22 *)
```

BCH

$$\text{LS}\left[\overline{x} + \overline{y}, \ \frac{\overline{x\,y}}{2}, \ \frac{1}{12}\overline{x\,\overline{x\,y}} + \frac{1}{12}\overline{\overline{x\,y}\,y}, \ \frac{1}{24}\overline{x\,\overline{\overline{x\,y}\,y}},\right.$$

$$-\frac{1}{720}\overline{x\,x\,\overline{x\,\overline{x\,y}}} + \frac{1}{180}\overline{x\,x\,\overline{\overline{x\,y}\,y}} + \frac{1}{180}\overline{x\,\overline{\overline{x\,y}\,y}\,y} + \frac{1}{120}\overline{\overline{x\,y}\,\overline{\overline{x\,y}\,y}} + \frac{1}{360}\overline{\overline{x\,\overline{x\,y}}\,\overline{x\,y}} - \frac{1}{720}\overline{\overline{\overline{x\,y}\,y}\,y\,y},$$

$$-\frac{x\,x\,x\,\overline{\overline{x\,y}\,y}}{1440} + \frac{1}{360}\overline{x\,x\,\overline{\overline{x\,y}\,y}\,y} + \frac{1}{240}\overline{x\,\overline{x\,y}\,\overline{\overline{x\,y}\,y}} + \frac{1}{720}\overline{x\,\overline{x\,\overline{x\,y}}\,\overline{x\,y}} - \frac{x\,\overline{\overline{x\,y}\,y}\,y\,y}{1440}, \ \dots\left.\right]$$

KVDirect
```
{F = LS[{x, y}, Fs], G = LS[{x, y}, Gs]}; Fs["y"] = 1/2;
SeriesSolve[{F, G},
```
$\hbar^{-1}$ (LS[x + y] - BCH[y, x] $\equiv$ F - G - Ad[-x][F] + Ad[y][G]) $\bigwedge$ div$_x$[F] + div$_y$[G] $\equiv$
$\frac{1}{2}$ tr$_u$[adSeries[$\frac{ad}{e^{ad} - 1}$, x][u] + adSeries[$\frac{ad}{e^{ad} - 1}$, y][u] - adSeries[$\frac{ad}{e^{ad} - 1}$, BCH[x, y]][u]]];
```
{F, G}  (* Can raise degree to 13 *)
```

KVDirect

$\Big\{$LS$\Big[\frac{\overline{y}}{2}, \frac{\overline{xy}}{6}, \frac{1}{24}\overline{xyy}, -\frac{1}{180}\overline{x\,x\overline{xy}} + \frac{1}{80}\overline{x\overline{xyy}} + \frac{1}{360}\overline{\overline{xy}yy},$

$-\frac{1}{720}\overline{x\,x\overline{xyy}} + \frac{1}{240}\overline{x\overline{xyy}y} + \frac{1}{240}\overline{\overline{xy}\overline{xyy}} + \frac{1}{720}\overline{\overline{xxy}\overline{xy}} - \frac{\overline{\overline{xy}yy}}{1440}, \frac{\overline{xxx\overline{xy}}}{5040} - \frac{\overline{xxx\overline{xyy}}}{1344} +$

$\frac{13\,\overline{xx\overline{xyy}y}}{15\,120} + \frac{1}{840}\overline{x\overline{xy}\overline{xyy}} + \frac{\overline{x\overline{xxy}\overline{xy}}}{3360} + \frac{\overline{x\overline{xyy}yy}}{6720} + \frac{\overline{\overline{xy}\overline{xyy}y}}{1260} + \frac{\overline{x\overline{xyy}\overline{xy}}}{1680} - \frac{\overline{\overline{xy}yyy}}{10\,080}, \ldots\Big],$

LS$\Big[0, \frac{\overline{xy}}{12}, \frac{1}{24}\overline{xyy}, -\frac{1}{360}\overline{x\,x\overline{xy}} + \frac{1}{120}\overline{x\overline{xyy}} + \frac{1}{180}\overline{\overline{xy}yy},$

$-\frac{1}{720}\overline{x\,x\overline{xyy}} + \frac{1}{240}\overline{x\overline{xyy}y} + \frac{1}{240}\overline{\overline{xy}\overline{xyy}} + \frac{1}{720}\overline{\overline{xxy}\overline{xy}} - \frac{\overline{\overline{xy}yy}}{1440}, \frac{\overline{xxx\overline{xy}}}{10\,080} - \frac{\overline{xxx\overline{xyy}}}{2016} +$

$\frac{\overline{xx\overline{xyy}y}}{1890} + \frac{\overline{x\overline{xy}\overline{xyy}}}{1120} + \frac{\overline{x\overline{xxy}\overline{xy}}}{5040} + \frac{\overline{x\overline{xyy}yy}}{2520} + \frac{1}{840}\overline{\overline{xy}\overline{xyy}y} + \frac{\overline{x\overline{xyy}\overline{xy}}}{1260} - \frac{\overline{\overline{xy}yyy}}{5040}, \ldots\Big]\Big\}$

RandomOps
```
{b[F, G], tr_x[F]}
```

RandomOps

$\Big\{$LS$\Big[0, 0, -\frac{1}{24}\overline{xyy}, -\frac{1}{48}\overline{xyy\,y}, \frac{1}{720}\overline{x\,x\overline{xyy}} - \frac{1}{240}\overline{x\overline{xyy}y} - \frac{\overline{\overline{xy}\overline{xy}}}{1440} - \frac{1}{720}\overline{x\overline{xy}\overline{xy}} - \frac{1}{360}\overline{\overline{xy}yyy},$

$\frac{\overline{xx\overline{xyy}y}}{1440} - \frac{1}{480}\overline{x\overline{xyy}yy} - \frac{1}{288}\overline{\overline{xy}\overline{xyy}y} - \frac{7\,\overline{x\overline{xyy}\overline{xy}}}{2880} + \frac{\overline{\overline{xy}yyy}}{2880}, \ldots\Big],$

CWS$\Big[-\frac{\widehat{y}}{6}, \frac{\widehat{yy}}{24}, \frac{\widehat{xxy}}{180} + \frac{\widehat{xyy}}{80} - \frac{\widehat{yyy}}{360}, -\frac{\widehat{xxyy}}{180} + \frac{\widehat{xyxy}}{240} - \frac{\widehat{xyyy}}{240} - \frac{\widehat{yyyy}}{1440},$

$-\frac{\widehat{xxxxy}}{5040} + \frac{\widehat{xxxyy}}{6720} - \frac{\widehat{xxyxy}}{1120} + \frac{2\,\widehat{xxyyy}}{945} - \frac{\widehat{xyxyy}}{336} + \frac{\widehat{xyyyy}}{6720} + \frac{\widehat{yyyyy}}{10\,080}, \frac{\widehat{xxxxyy}}{3360} - \frac{\widehat{xxxyxy}}{1344} - \frac{\widehat{xxxyyy}}{2240} +$

$\frac{\widehat{xxyxxy}}{2016} + \frac{13\,\widehat{xxyxyy}}{10\,080} + \frac{\widehat{xxyyxy}}{1680} - \frac{\widehat{xxyyyy}}{3780} - \frac{\widehat{xyxyxy}}{840} + \frac{\widehat{xyxyyy}}{5040} + \frac{\widehat{xyyxyy}}{2240} + \frac{\widehat{xyyyyy}}{6720} + \frac{\widehat{yyyyyy}}{60\,480}, \ldots\Big]\Big\}$

VCapSolution
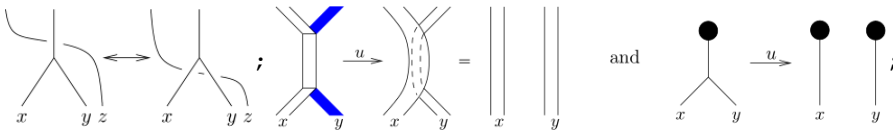

```
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
UnitarityEqn = (V ** (V // dA) ≡ Es[⟨x → LS[0], y → LS[0]⟩, CWS[0]]);
CapEqn = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡ (Cap (Cap // dσ[x, y]) // dc[x] // dc[y]));
βs["x"] = 1/2; βs["y"] = 0;
SeriesSolve[{α, β, γ, κ}, (ℏ⁻¹ R4Eqn) ⋀ UnitarityEqn ⋀ CapEqn];
{V, κ}
```

VCapSolution

     SeriesSolve::ArbitrarilySetting : In degree 1 arbitrarily setting {κs[x] → 0}.

VCapSolution

     SeriesSolve::ArbitrarilySetting : In degree 3 arbitrarily setting {αs[x, y, y] → 0}.

VCapSolution

     SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {αs[x, x, x, y, y] → 0}.

VCapSolution

     General::stop : Further output of SeriesSolve::ArbitrarilySetting will be suppressed during this calculation. ≫

VCapSolution

atkv

```
logF = Λ[V]〚1〛 // dσ[{x, y} → {y, x}];

logF // EulerE // adSeries[(e^ad - 1)/ad, logF, tb]
```

atkv

$$\Big\langle \overline{x} \to \mathrm{LS}\Big[\frac{\overline{y}}{2},\ \frac{\overline{x\,y}}{6},\ \frac{1}{24}\overline{\overline{x\,y}\,y},\ -\frac{1}{180}\overline{x\,\overline{x\,\overline{x\,y}}} + \frac{1}{80}\overline{x\,\overline{\overline{x\,y}\,y}} + \frac{1}{360}\overline{\overline{\overline{x\,y}\,y}\,y},$$

$$-\frac{1}{720}\overline{x\,x\,\overline{\overline{x\,y}\,y}} + \frac{1}{240}\overline{x\,\overline{\overline{\overline{x\,y}\,y}\,y}} + \frac{1}{240}\overline{\overline{x\,y}\,\overline{\overline{x\,y}\,y}} + \frac{1}{720}\overline{\overline{x\,\overline{x\,y}}\,\overline{x\,y}} - \frac{\overline{\overline{\overline{\overline{x\,y}\,y}\,y}\,y}}{1440},\ \frac{\overline{x\,x\,x\,\overline{x\,\overline{x\,y}}}}{5040} - \frac{\overline{x\,x\,x\,\overline{\overline{x\,y}\,y}}}{1344} +$$

$$\frac{13\,\overline{x\,x\,\overline{\overline{\overline{x\,y}\,y}\,y}}}{15\,120} + \frac{1}{840}\overline{x\,\overline{x\,y}\,\overline{\overline{x\,y}\,y}} + \frac{\overline{x\,\overline{x\,\overline{x\,y}}\,\overline{x\,y}}}{3360} + \frac{\overline{x\,\overline{\overline{\overline{x\,y}\,y}\,y}\,y}}{6720} + \frac{\overline{\overline{x\,y}\,\overline{\overline{\overline{x\,y}\,y}\,y}}}{1260} + \frac{\overline{x\,\overline{\overline{\overline{x\,y}\,y}\,y}\,\overline{x\,y}}}{1680} - \frac{\overline{\overline{\overline{\overline{\overline{x\,y}\,y}\,y}\,y}\,y}}{10\,080},\ \dots\Big],$$

$$\overline{y} \to \mathrm{LS}\Big[0,\ \frac{\overline{x\,y}}{12},\ \frac{1}{24}\overline{\overline{x\,y}\,y},\ -\frac{1}{360}\overline{x\,\overline{x\,\overline{x\,y}}} + \frac{1}{120}\overline{x\,\overline{\overline{x\,y}\,y}} + \frac{1}{180}\overline{\overline{\overline{x\,y}\,y}\,y},$$

$$-\frac{1}{720}\overline{x\,x\,\overline{\overline{x\,y}\,y}} + \frac{1}{240}\overline{x\,\overline{\overline{\overline{x\,y}\,y}\,y}} + \frac{1}{240}\overline{\overline{x\,y}\,\overline{\overline{x\,y}\,y}} + \frac{1}{720}\overline{\overline{x\,\overline{x\,y}}\,\overline{x\,y}} - \frac{\overline{\overline{\overline{\overline{x\,y}\,y}\,y}\,y}}{1440},\ \frac{\overline{x\,x\,x\,\overline{x\,\overline{x\,y}}}}{10\,080} - \frac{\overline{x\,x\,x\,\overline{\overline{x\,y}\,y}}}{2016} +$$

$$\frac{\overline{x\,x\,\overline{\overline{\overline{x\,y}\,y}\,y}}}{1890} + \frac{\overline{x\,\overline{x\,y}\,\overline{\overline{x\,y}\,y}}}{1120} + \frac{\overline{x\,\overline{x\,\overline{x\,y}}\,\overline{x\,y}}}{5040} + \frac{\overline{x\,\overline{\overline{\overline{x\,y}\,y}\,y}\,y}}{2520} + \frac{1}{840}\overline{\overline{x\,y}\,\overline{\overline{\overline{x\,y}\,y}\,y}} + \frac{\overline{x\,\overline{\overline{\overline{x\,y}\,y}\,y}\,\overline{x\,y}}}{1260} - \frac{\overline{\overline{\overline{\overline{\overline{x\,y}\,y}\,y}\,y}\,y}}{5040},\ \dots\Big]\Big\rangle$$

Associator

```
Φs[2, 1] = Φs[3, 1] = Φs[3, 2] = 0; Φs[3, 1, 2] = 1/24; Φ = DKS[3, Φs];

SeriesSolve[Φ, (Φ^σ[3,2,1] ≡ -Φ) ⋀ (Φ ** Φ^σ[1,23,4] ** Φ^σ[2,3,4] ≡ Φ^σ[12,3,4] ** Φ^σ[1,2,34])];

Φ (* Can raise degree to 10 *)
```

Associator

SeriesSolve::ArbitrarilySetting : In degree 3 arbitrarily setting {Φs[3, 1, 1, 2] → 0}.

Associator

SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {Φs[3, 1, 1, 1, 1, 2] → 0}.

Associator

$$\mathrm{DKS}\Big[0,\ \frac{1}{24}\overline{t_{13}\,t_{23}},\ 0,\ -\frac{7\,\overline{\overline{t_{13}\,t_{23}}\,t_{23}\,t_{23}}}{5760} + \frac{7\,\overline{t_{13}\,\overline{\overline{t_{13}\,t_{23}}\,t_{23}}}}{5760} - \frac{\overline{t_{13}\,t_{13}\,\overline{t_{13}\,t_{23}}}}{1440},$$

$$0,\ \frac{31\,\overline{\overline{\overline{t_{13}\,t_{23}}\,t_{23}\,t_{23}}\,t_{23}\,t_{23}}}{967\,680} - \frac{157\,\overline{t_{13}\,\overline{\overline{t_{13}\,t_{23}}\,t_{23}\,t_{23}}\,\overline{t_{13}\,t_{23}}}}{1\,935\,360} - \frac{31\,\overline{t_{13}\,t_{23}\,\overline{\overline{t_{13}\,t_{23}}\,t_{23}\,t_{23}}}}{387\,072} -$$

$$\frac{31\,\overline{t_{13}\,\overline{\overline{t_{13}\,t_{23}}\,t_{23}\,t_{23}}\,t_{23}}}{483\,840} + \frac{11\,\overline{t_{13}\,\overline{t_{13}\,\overline{t_{13}\,t_{23}}\,t_{13}\,t_{23}}}}{290\,304} + \frac{31\,\overline{t_{13}\,\overline{\overline{t_{13}\,t_{23}}\,\overline{t_{13}\,t_{23}}\,t_{23}}}}{725\,760} +$$

$$\frac{83\,\overline{t_{13}\,\overline{t_{13}\,\overline{\overline{t_{13}\,t_{23}}\,t_{23}}\,t_{23}}}}{967\,680} - \frac{13\,\overline{t_{13}\,\overline{t_{13}\,\overline{t_{13}\,\overline{t_{13}\,t_{23}}}\,t_{23}}}}{241\,920} + \frac{\overline{t_{13}\,\overline{t_{13}\,\overline{t_{13}\,\overline{t_{13}\,\overline{t_{13}\,t_{23}}}}}}}{60\,480},\ \dots\Big]$$

TheBuckle

$R = DKS\big[t[1, 2]\big/2\big];$

$Z_B = (-\Phi)^{\sigma[13,2,4]} ** \Phi^{\sigma[1,3,2]} ** R^{\sigma[2,3]} ** (-\Phi)^{\sigma[1,2,3]} ** \Phi^{\sigma[12,3,4]};$

$Z_B @ \{4\}$

TheBuckle

$$DKS\Big[\frac{\overline{t_{23}}}{2}, -\frac{1}{12}\overline{t_{13}\,t_{23}} - \frac{1}{24}\overline{t_{14}\,t_{24}} + \frac{1}{24}\overline{t_{14}\,t_{34}} + \frac{1}{12}\overline{t_{24}\,t_{34}}, 0,$$

$$\frac{\overline{\overline{t_{13}\,t_{23}}\,t_{23}\,t_{23}}}{5760} + \frac{7\,\overline{\overline{t_{14}\,t_{24}}\,t_{24}\,t_{24}}}{5760} + \frac{\overline{\overline{t_{14}\,t_{34}}\,t_{24}\,t_{24}}}{1920} - \frac{\overline{\overline{t_{14}\,t_{34}}\,t_{34}\,t_{24}}}{1920} - \frac{7\,\overline{\overline{t_{14}\,t_{34}}\,t_{34}\,t_{34}}}{5760} -$$

$$\frac{\overline{\overline{t_{24}\,t_{34}}\,t_{34}\,t_{34}}}{5760} + \frac{\overline{t_{14}\,\overline{t_{24}\,t_{34}}\,t_{24}}}{1920} + \frac{\overline{t_{14}\,t_{24}\,\overline{t_{14}\,t_{34}}}}{1920} - \frac{\overline{t_{14}\,t_{34}\,\overline{t_{24}\,t_{34}}}}{1920} - \frac{1}{720}\overline{t_{13}\,\overline{t_{13}\,t_{23}}\,t_{23}} +$$

$$\frac{1}{720}\overline{t_{13}\,t_{13}\,\overline{t_{13}\,t_{23}}} - \frac{7\,\overline{t_{14}\,\overline{t_{14}\,t_{24}}\,t_{24}}}{5760} + \frac{7\,\overline{t_{14}\,\overline{t_{14}\,t_{34}}\,t_{34}}}{5760} - \frac{\overline{t_{14}\,\overline{t_{24}\,t_{34}}\,t_{34}}}{5760} + \frac{\overline{t_{14}\,t_{14}\,\overline{t_{14}\,t_{24}}}}{1440} -$$

$$\frac{\overline{t_{14}\,\overline{t_{14}\,t_{14}}\,t_{34}}}{1440} - \frac{1}{960}\overline{t_{14}\,t_{14}\,\overline{t_{24}\,t_{34}}} + \frac{\overline{t_{14}\,t_{24}\,\overline{t_{24}\,t_{34}}}}{5760} - \frac{1}{960}\overline{t_{24}\,\overline{t_{24}\,t_{34}}\,t_{34}} - \frac{\overline{t_{24}\,t_{24}\,\overline{t_{24}\,t_{34}}}}{5760}, \ldots\Big]$$
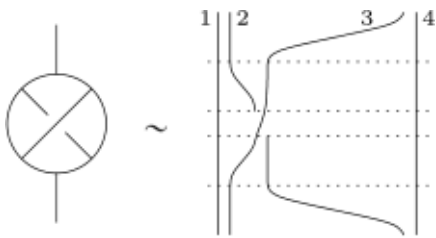
VFromPhi

$\big(\mathrm{El}[Z_B\ /\!/\ \alpha\mathrm{Map}[1, 2, 3, 4],\ \mathrm{CWS}[0]]\ /\!/\ \Gamma\ /\!/\ t\eta^1\ /\!/\ t\eta^3\ /\!/\ h\eta^2\ /\!/\ h\eta^4\ /\!/\ h\sigma[\{3\} \to \{2\}]\ /\!/$
$\quad t\sigma[\{2, 4\} \to \{1, 2\}]\big)[\![1]\!]$

VFromPhi

$$\Big\langle 1 \to \mathrm{LS}\Big[0, -\frac{\overline{1\,2}}{24}, 0, \frac{7\,\overline{1\,\overline{1\,2}}}{5760} - \frac{7\,\overline{1\,\overline{1\,2}\,2}}{5760} + \frac{\overline{\overline{1\,2}\,2\,2}}{1440}, 0, -\frac{31\,\overline{1\,1\,1\,\overline{1\,2}}}{967\,680} + \frac{31\,\overline{1\,1\,1\,\overline{1\,2}\,2}}{483\,840} - \frac{83\,\overline{1\,1\,\overline{1\,2}\,2\,2}}{967\,680} -$$

$$\frac{31\,\overline{1\,\overline{1\,2}\,\overline{1\,2}\,2}}{725\,760} - \frac{31\,\overline{1\,1\,\overline{1\,2}\,\overline{1\,2}}}{645\,120} + \frac{13\,\overline{1\,\overline{1\,2}\,2\,2\,2}}{241\,920} + \frac{101\,\overline{\overline{1\,2}\,\overline{1\,2}\,2\,2}}{1\,451\,520} + \frac{527\,\overline{1\,\overline{1\,2}\,2\,\overline{1\,2}}}{5\,806\,080} - \frac{\overline{\overline{1\,2}\,2\,2\,2\,2}}{60\,480}, \ldots\Big],$$

$$2 \to \mathrm{LS}\Big[\frac{\overline{1}}{2}, -\frac{\overline{1\,2}}{12}, 0, \frac{\overline{1\,\overline{1\,2}}}{5760} - \frac{1}{720}\overline{1\,\overline{1\,2}\,2} + \frac{1}{720}\overline{\overline{1\,2}\,2\,2}, -\frac{\overline{1\,1\,\overline{1\,2}}}{7680} + \frac{11\,\overline{1\,1\,\overline{1\,2}\,2}}{3840} - \frac{\overline{\overline{1\,2}\,\overline{1\,2}}}{6912},$$

$$-\frac{1\,1\,1\,\overline{1\,\overline{1\,2}}}{645\,120} + \frac{23\,\overline{1\,1\,1\,\overline{1\,2}\,2}}{483\,840} - \frac{13\,\overline{1\,1\,\overline{1\,2}\,2\,2}}{161\,280} - \frac{\overline{1\,\overline{1\,2}\,\overline{1\,2}\,2}}{22\,680} -$$

$$\frac{41\,\overline{1\,1\,\overline{1\,2}\,\overline{1\,2}}}{580\,608} + \frac{1\,\overline{\overline{1\,2}\,2\,2\,2}}{15\,120} + \frac{\overline{\overline{1\,2}\,\overline{1\,2}\,2\,2}}{12\,096} + \frac{71\,\overline{1\,\overline{1\,2}\,2\,\overline{1\,2}}}{483\,840} - \frac{\overline{\overline{1\,2}\,2\,2\,2\,2}}{30\,240}, \ldots\Big]\Big\rangle$$

Borromean

```mathematica
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
iRs[a_, b_] := Es[⟨a → LS[0], b → -LS[LW@a]⟩, CWS[0]];
ξ = iRs[r, 6] Rs[2, 4] iRs[g, 9] Rs[5, 7] iRs[b, 3] Rs[8, 1];
```



```mathematica
Do[ξ = ξ // dm[r, k, r], {k, 1, 3}];
Do[ξ = ξ // dm[g, k, g], {k, 4, 6}];
Do[ξ = ξ // dm[b, k, b], {k, 7, 9}];
{ξ〚1〛ᵣ@{5}, ξ〚2〛@{5}} // Print
```
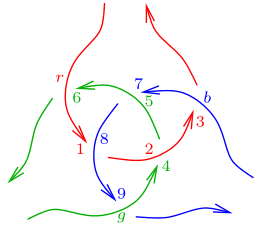
Borromean

$$\Big\{ \mathrm{LS}\Big[0, \overline{bg}, \tfrac{1}{2}\overline{b\,\overline{bg}} + \overline{b\,\overline{gr}} + \tfrac{1}{2}\overline{\overline{bg}\,g}, \tfrac{1}{6}\overline{b\,b\,\overline{bg}} + \tfrac{1}{2}\overline{b\,\overline{b\,\overline{gr}}} + \tfrac{1}{2}\overline{b\,\overline{g\,\overline{gr}}} + \tfrac{1}{4}\overline{b\,\overline{\overline{bg}\,g}} + \tfrac{1}{2}\overline{b\,\overline{\overline{gr}\,r}} + \tfrac{1}{6}\overline{\overline{\overline{bg}\,g}\,g},$$

$$\tfrac{1}{24}\overline{bb\,\overline{b\,\overline{bg}}} + \tfrac{1}{6}\overline{bb\,\overline{b\,\overline{gr}}} + \tfrac{1}{4}\overline{bb\,\overline{g\,\overline{gr}}} + \tfrac{1}{12}\overline{bb\,\overline{\overline{bg}\,g}} + \tfrac{1}{4}\overline{bb\,\overline{\overline{gr}\,r}} + \tfrac{1}{6}\overline{bg\,\overline{g\,\overline{gr}}} + \tfrac{1}{4}\overline{bg\,\overline{\overline{gr}\,r}} - \overline{b\,\overline{\overline{gr}\,g}} +$$

$$\tfrac{1}{12}\overline{b\,\overline{\overline{bg}\,g}\,g} - 2\overline{b\,\overline{brg}\,g} + \tfrac{1}{6}\overline{b\,\overline{\overline{gr}\,r}\,r} + \tfrac{1}{2}\overline{\overline{bg}\,\overline{b\,\overline{gr}}} - \overline{\overline{bg}\,\overline{brg}} - \tfrac{1}{12}\overline{\overline{\overline{bbg}}\,\overline{bg}} - \tfrac{1}{2}\overline{\overline{\overline{bg}\,r}\,\overline{gr}} + \tfrac{1}{24}\overline{\overline{\overline{bg}\,g}\,g\,g}, \dots\Big],$$

$$\mathrm{CWS}\Big[0, 0, 2\overline{bgr}, \overline{bbgr} - \overline{bgbr} + \overline{bggr} - \overline{bgrg} + \overline{bgrr} - \overline{brgr}, \tfrac{\overline{bbbgr}}{3} - \tfrac{\overline{bbgbr}}{2} + \tfrac{\overline{bbggr}}{2} + \tfrac{\overline{bbgrg}}{2} + \tfrac{\overline{bbgrr}}{2} + \tfrac{\overline{bbrbg}}{2} -$$

$$\tfrac{3\,\overline{bbrgr}}{2} + \tfrac{\overline{bgbrr}}{2} - \tfrac{3\,\overline{bggbr}}{2} + \tfrac{\overline{bgggr}}{3} - \tfrac{\overline{bggrg}}{2} + \tfrac{\overline{bggrr}}{2} + \tfrac{\overline{bgrgg}}{2} - \tfrac{3\,\overline{bgrrg}}{2} + \tfrac{\overline{bgrrr}}{3} + \tfrac{\overline{brggr}}{2} - \tfrac{\overline{brgrr}}{2} + \tfrac{\overline{brrgr}}{2}, \dots\Big]\Big\}$$

The below is adapted from pensieve://Projects/WKO4/CellExport.nb.

```mathematica
ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
   DeleteFile[temp],
   (* else *) Print["Exporting " <> fname <> "..."];
   If[exists, DeleteFile[fname]];
   RenameFile[temp, fname]
  ];
  fname
 ]


SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
TagProperties[_] := {};
TagProperties["Initialization"] = {PageWidth → 8};
TagProperties["VCapSolution"] = {PageWidth → 7};
TagProperties["Borromean"] = {PageWidth → 6.6};
Options[CellExport] = {
   PageWidth → 6.28, CellFilter → Identity, ExportBaseFilename → Automatic,
   ExportFormat → ".pdf", ExportOptions → {}, Split → False
  };
CellExport[tag_String, opts___Rule] := CellExport[
   NotebookGet[EvaluationNotebook[]],
   tag, opts
  ];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, opts___Rule] := Module[
```

```
    {cells, cell, filename, format},
    filename = OptionValue[ExportBaseFilename] /. Automatic → tag;
    format = OptionValue[ExportFormat];
    cells = OptionValue[CellFilter][Cases[
        nb, c_Cell /; FreeQ[List@@c, Cell] && ! FreeQ[c, CellTags → tag],
        Infinity
      ]];
    If[! OptionValue[Split],
      If[Length[cells] ≥ 1,
        If[Length[cells] == 1,
          cells = Append[First[cells], PageWidth → 72 OptionValue[PageWidth]],
          cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
        ];
        ConditionalExport[
          filename <> format, cells,
          ImageResolution → 300,
          OptionValue[ExportOptions]
        ]
      ],
      k = 0;
      Table[
        ++k;
        ConditionalExport[
          filename <> "-" <> ToString[k] <> format,
          Append[cell, PageWidth → 72 OptionValue[PageWidth]],
          ImageResolution → 300,
          OptionValue[ExportOptions]
        ],
        {cell, cells}
      ]
    ]
  ];
nb = NotebookGet[EvaluationNotebook[]];
tags = Cases[nb, (CellTags → tag_) :> tag, Infinity] // Union;
CellExport /@ tags
```

Exporting Borromean.pdf...

{Associator.pdf, atkv.pdf, BCH.pdf, Borromean.pdf, Initialization.pdf,
 KVDirect.pdf, RandomOps.pdf, TheBuckle.pdf, VCapSolution.pdf, VFromPhi.pdf}