This should be a footnote.

On Raoul Bott's "On Invariants of Manifold"

Dror Bar-Natan

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I'm not sure how to introduce a review paper [B]. So rather than commenting on the paper as whole, I will concentrate on my subjective view of just one paragraph—a paragraph which I think I influenced and which ended up influencing me very deeply. A paragraph I am sure Raoul was uncomfortable writing, for at the time he was uncomfortable with his understanding of the underlying mathematics as I have explained it to him [BN1]—uncomfortable enough to later rewrite (with Taubes) this bit of mathematics in his own language [BT], making my own work completely obsolete.

Given the removal of [BN2], perhaps [BN1] should be [BN]?

Raoul starts with a beautiful review of the Euler characteristic, Pontryagin numbers, and the indices of elliptic operators. Everything is so smooth and flowing one may almost mistake things to be trivial. Then, on page 37, comes the paragraph I wish to discuss:

> . . . In attempting to carry out the finite dimensional program in this infinite dimensional context, one encounters all the road blocks which over the years the field theorists have learned to overcome. Indeed the Hessian of S_M at μ turns out to be degenerate and the Fadeev Popof procedures have to be applied. For this purpose an auxiliary Riemann structure, g, on M has to be chosen, and once this choice is made, $\alpha_l(M,\mu)$ is seen to make sense, but as a very complicated integral involving the Green's operator of the Laplacian of g. At this stage one has to show that this integral is independent of the choice of g, and the physicists have developed formal procedures to show this independence—called "B.R.S. invariance". However it is only the recent work of Dror Bar-Natan [BN1], and Axelrod & Singer [AS] that brings these questions into proper mathematical form.
> . . .

Should be g!

The beauty and smoothness are gone. Instead we are instructed to follow the rules, as dictated by the greats that are above us: choose a metric g, write "very complicated integrals", and follow "formal procedures".

That was my fault! For the whole year before, in many meetings with Raoul, I have repeatedly explained my thesis to him ("it's so simple, you just follow the Feynman-Faddeev-Popov-Becchi-Rouet-Stora rules, what's here not to understand?"), and he repeatedly refused to understand. The quoted paragraph must have been written when he temporarily surrendered.

But soon after, his instincts won. The question was a question in topology and the

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AQ1

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"physics" resolution seemed to him to be too complicated, using tools that seemed inappropriate for the subject matter. And indeed along with Taubes, Raoul was able to reduce the "complicated integrals" to the integrals of pullbacks of volume forms of spheres to some configuration spaces of points on M, and the lengthy "formal procedures" became a simple application of a fiber-wise Stokes' theorem (see [BT], and a more complete exposition by D. Thurston, [Th]). My earlier work became obsolete.

I learned something from this story. Something about simplicity—about what tools are appropriate for what problem, and about how hard one should work, and how beneficial it is, to find the "right" answer to a question, rather than just an answer that works. I still can't quite quantify what I've learned—perhaps it is impossible—yet it had been guiding me ever since.

References

[AS] S. Axelrod and I. M. Singer, *Chern-Simons Perturbation Theory,* Proc. XXth DGM Conference (New York, 1991) (S. Catto and A. Rocha, eds.) World Scientific, 1992, 3–45. arXiv: hep-th/9110056.

[BN1] D. Bar-Natan, *Perturbative aspects of the Chern-Simons topological quantum field theory*, Ph.D. thesis, Princeton Univ., June 1991.

Delete

[BN2] D. Bar-Natan, *On the Vassiliev knot invariants,* Topology **34** (1995) 423–472. AQ2

[B] R. Bott, *On Invariants of Manifolds,* in *Modern Methods in Complex Analysis (Princeton, NJ, 1992)*, Ann. of Math. Stud. **137** Princeton Univ. Press (1995) 29–39.

[BT] R. Bott and C. Taubes, *On the self-linking of knots,* Jour. Math. Phys. **35** (1994).

[Th] D. Thurston, *Integral expressions for the Vassiliev knot invariants,* Harvard University senior thesis, April 1995, arXiv: math.QA/9901110.

AUTHOR QUERIES

AQ1. Please check if author affiliation is okay. Yes.

AQ2. Please cite Ref. [BN2] in text. Delete Ref. [DN2].