van der Veen on Quantum Coinvariants

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Really, "Quantum Invariants Theory"

Understanding Quantum invariants as a theory of invariants of quantum groups.

$\boxed{yx = q xy \text{ "the quantum plane"}}$

- What is TQFT?
- Is there a recursion for colored HOMFLY?
- What about $\mathfrak{X}$ beyond ADE?
- What if you can't afford an iPhone 6? Buy a quantum group!

<u>Plan</u>: ① Finding quantum $SL(2,\mathbb{C})$ from the quantum plane.

② $\mathbb{C}_q M_{Flat}$ "quantized flat connections"

③ FRT construction of matrix quantum groups.

Faddeev, Reshetikhin, Takhtajan?

④ Colored HOMFLY, Ising model. 0:15

I Constructing quantum $SL(2,\mathbb{C})$ from $qyx = xy$

$$\mathbb{C}_q^2 = \{yx = qxy\}$$

Frenkel-Kim: Quantum Teich space from quantum plane using modular double.

$SL_q(2,\mathbb{C})$ is the group of linear symmetries of $\mathbb{C}_q^2$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$$

(x & y commute w/ a,b,c,d)

s.t. $(cx+dy)(ax+by) = q(ax+by)(cx+dy)$

$$\Rightarrow \quad ca = qac \qquad db = qbd$$

$$cb + qda = q^2bc + qad$$

Also require

$$(x \;\; y)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (ax+cy \quad bx+dy)$$

is also a symmetry

$\rightsquigarrow$

$$SL_q(2,\mathbb{C}) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : \begin{array}{ll} ca = qac & db = qbd \\ ba = qab & ad - q^{-1}bc = \end{array}\right)$$

$$SL_q(2,\mathbb{C}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \begin{matrix} ba = qab \\ dc = qcd \\ bc = cb \end{matrix} \quad \begin{matrix} ad - q^{-1}bc = \\ da - qbc \end{matrix} \right\}$$

$$\det_q = ad - q^{-1}bc = da - qbc$$

These relations can be rephrased using the R-matrix:

$$R = \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & q-q^{-1} & 0 \\ 0 & 0 & 0 & q \end{pmatrix}$$

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A \otimes A = \left(\begin{array}{cc|c} aa & ab & \cdots \\ ac & ad & \\ \hline ca & cb & \\ cc & cd & \cdots \end{array}\right)$

$$(A \otimes A) R = R (A \otimes A)$$

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