

# van der Veen on Quantum Coinvariants

July-29-15 8:46 AM

Really, "Quantum Invariants Theory"

Understanding Quantum invariants as a theory of invariants of quantum groups.

$yx = qxy$   
"the quantum plane"

- What is TQFT?

- Is there a recursion for colored HOMFLY?

- What about  $\mathcal{X}$  beyond ADE?

- What if you can't afford an iPhone 6?  
Buy a quantum group!

Plan: ① Finding quantum  $SL(2, \mathbb{C})$  from the quantum plane.

②  $\mathbb{C}_q M_{Flat}$  "quantized flat connections"

③ FRT construction of matrix quantum groups.  
↑ Frenkel  
↓ Takhtajan?  
Reshetikhin

④ colored HOMFLY, Ising model.

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① Constructing quantum  $SL(2, \mathbb{C})$  from

$$qyx = xy$$

$$\mathbb{C}_q^2 = \{yx = qxy\}$$

Frenkel-Kim:  
Quantum Teich space  
From quantum planes  
Using modular double.

$SL_q(2, \mathbb{C})$  is the group of linear symmetries  
of  $\mathbb{C}_q^2$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix} \quad \left( \begin{array}{l} x, y \\ \text{commute w/} \\ a, b, c, d \end{array} \right)$$

s.t.  $(cx + dy)(ax + by) = q(ax + by)(cx + dy)$

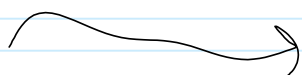
$$\Rightarrow ca = qac \quad db = qbd$$

$$cb + qda = q^2bc + qad$$

Also require

$$(cx + dy) \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ax + cy & bx + dy \end{pmatrix}$$

is also a symmetry



$$SL_q(2, \mathbb{C}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \begin{array}{ll} ca = qac & db = qbd \\ ba = qab & ad - q^{-1}bc = 1 \end{array} \right\}$$

$$SL_q(2, \mathbb{C}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\} : \left. \begin{aligned} ba &= qab & ad - q^{-1}bc &= \\ dc &= qcd & da - qbc &= \\ & & bc &= cb \end{aligned} \right\}$$

$$\det_q = ad - q^{-1}bc = da - qbc$$

These relations can be rephrased using the R-matrix:

$$R = \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & q - q^{-1} & 0 \\ 0 & 0 & 0 & q \end{pmatrix}$$

IF  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  then  $A \otimes A = \begin{pmatrix} aa & ab & | & - \\ ac & ad & | & - \\ \hline ca & cb & | & - \\ cc & cd & | & - \end{pmatrix}$

$$(A \otimes A) R = R (A \otimes A)$$

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②