Thang Le on the unified quantum invariant of integral homology 3-spheres associated to simple Lie algebras

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1. History & survey
2. Habiro ring
3. Main result
4. Construction of the univ. invts.

See also 2014-07

1984 Jones poly.

* generalizations: $\mathfrak{g}$ simple Lie algebra,

L: m-component framed oriented link in $\mathbb{R}^3/S^3$

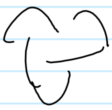
$V_1 \dots V_m$ reps of $\mathfrak{g}$.

$$\longrightarrow J_L^{\mathfrak{g}}(V_1 \dots V_m) \in \mathbb{Z}\left[q^{\pm\frac{1}{2D}}\right]$$

$$D = \det(\text{Cartan}(\mathfrak{g}))$$

If $\mathfrak{g} = sl_2$, all V_i's the defining, then this is Jones.

Example:

$K =$  $\quad J_K^{sl_2}(V_n) = [n] \sum_{k=0}^{n-1} q^{-kn} (\_\ \_\ \_)$

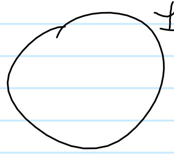
n-dim

$k=0$

$$(1-q^{1-n})\cdot(1-q^{2-n})\dots(1-q^{k-n})$$

Reshetikhin-Turaev: $\underbrace{\text{Surgery}}_{\text{Try to use}}$ on link,

kernel is Kirby moves:

1.  $^{\pm 1} \sim \phi$ 2. Kirby II

Witten 1988 TQFT etc.

$\rightsquigarrow$ A TQFT/modular category for any g at root of unity.

Get a 3-mfd invt for all g & some roots of unity

⋮

Habiro ring:

$$\widehat{\mathbb{Z}[q]} = \varprojlim \mathbb{Z}[q]/(q)_n \qquad (q)_n = \prod_{i=1}^{n}(1-q^i)$$

$$F = \sum_{n=0}^{\infty} f_n(q)(q)_n \qquad f_n \in \mathbb{Z}[q]$$

unique if $\deg(f_n) \leq n$.

if ξ is a root of 1, $\exists\ ev_\xi \in \mathbb{C}$

Properties

0:34.

Thm (Le, Habiro) For $\mathfrak{g}$ a simple Lie algebra, $M \in \mathbb{Z}HS$,

$$\exists!\ J_M^{\mathfrak{g}} \in \widehat{\mathbb{Z}[q]} \quad \text{s.t. } \forall \xi \in Z_{\mathfrak{g}}$$

roots of unity for $\mathfrak{g}$

$$ev_\xi(J_M^{\mathfrak{g}}) = J_M^{\mathfrak{g}}(\xi)$$

WRT invt.

Corollarys: 0. ?

1. Integrality.
2. Recovers LMO // $W^{\mathfrak{g}}$.