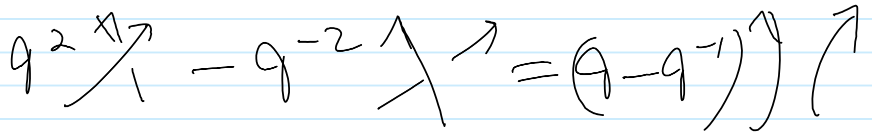
Thang Le in Qinhuangdao

July-13-15 11:11 PM

1984 Jones polynomial:

$$K \longrightarrow iV_L \in \mathbb{Z}[q^{\pm 1}]$$



$$\bigcirc \longrightarrow q + q^{-1}$$

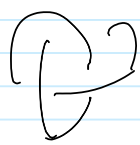
generalization via quantum groups

K knot, $\mathfrak{g}$ simple Lie alg, V f.d. rep.

$$J_K^{\mathfrak{g}}(V) \in \mathbb{Z}[q^{\pm 1}]$$

For $sl(2)$, $\mathbb{C}^2$, get old J_K.

$\forall n \geq 1$ $\exists V_n$ irred rep. of $sl(2)$, get $J_K(n)$, the colored Jones poly.

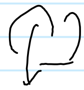
Example: The trefoil  $=K$

$$J_K(m) = [m]\, q^{2-m} \sum_{k=0}^{\infty} q^{-2km}(1-q^{2-2m}) \cdot (1-q^{4-2m}) \cdot \ldots \cdot (1-q^{2k-2m})$$

$\exists$ recurrence relation

$$a_2 J_K(m+2) + a_1 J_K(m+1) + a_0 J_K(m) = 0$$

w/ $a_2 = q^{2+10m} - q^{6m}$ $\quad a_1, a_0 \in \mathbb{Z}[q^{\pm m}, q^{\pm 1}]$

along w/ $J_K(0)=1$ & $J_K(1)=1$ this determines the coloured Jones of 

Two operators L, M acting on

$$\{f : \mathbb{Z} \to R = \mathbb{Z}[q, q^{-1}]\}$$

$$(Lf)(m) = f(m+1) \qquad LM = qML$$
$$(Mf)m = q^m f(m)$$

$$\mathbb{T} := R\langle M^{\pm 1}, L^{\pm}\rangle / (LM = qML)$$

$$\mathcal{A}_f := \{\alpha \in \mathbb{T} : \alpha f = 0\}$$

<u>Def</u> f is q-holonomic if $\mathcal{A}_f \neq 0$

For the trefoil, take $\langle \alpha \rangle = \mathcal{A}_f$

$$\alpha\big|_{q=1} = \underbrace{(M^4-1)}_{M\text{ factor}} \underbrace{(1+M^6L)(L-1)}_{A\text{-polynomial of}}$$

<u>Thm</u> (Garoufalidis, Le) $\forall K$ $J_K : \mathbb{N} \to R$ ($\mathbb{Z}$)

is q-holonomic.

is ...

$\rightsquigarrow \exists$ canonical $\alpha_K \in \mathbb{T}$

Conjecture (AJ conjecture, Garoufalidis, Frohman, Gelca, ?.)

$$\alpha_K\Big|_{q=1} = \binom{M-}{\text{factor}}(A-\text{poly}).$$

Le, Tran-Le: proven for for many 2-bridge knots & some pretzel knots, & torus knots.

On to sl_n. Reps parametrized by partitions $\lambda=(\lambda_1,\dots,\lambda_{n-1})$; get $J_K^{sl_n}(\lambda)\in R$

At $\lambda=\square$, get HOMFLYPT:

$$q^n \nearrow - q^{-n} \nwarrow = (q+q^{-1}))\,($$

$\longrightarrow \exists$ poly $W_K(\square) \in \mathbb{Z}[z^{\pm 1}, q^{\pm 1}]$ st. at $z=q^n$, get HOMFLYPT

This is true for all λ. First proof of this "sl_n-uniformization" by Morton.

Q ... (λ) is ... holonomic ...

Q $W_K(\lambda)$ is q-holonomic w.r.t. λ

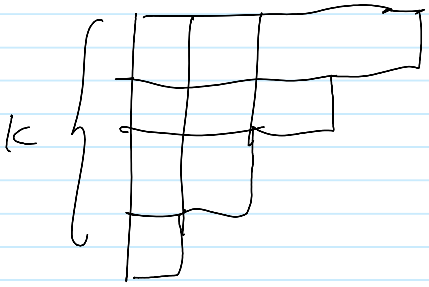
Namely, ...

Thm (Garoufalidis, Lauda, Le) $W_K(m\square)$ is q-holonomic.

proof uses quantum skew duality using Cautis-Kamnitzer-Morrison.

This can be generalized to partitions w/ a fixed number of rows/cols.

Q: the GLL techniques cannot do



$k \to \infty$.