Masbaum on An Application of TQFT to Modular Representation Theory

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k: a field.

$$Sp(2g,k)=\{M\in M_{2g\times 2g}(k) : MJ^tM=J\}$$

$$J=\begin{bmatrix}0 & I\\ -I & 0\end{bmatrix}$$

For $g=1$ get $sl(2,k)$.

Representations in char $=0$, say $k=\mathbb{C}$:

$$\text{Irreps}\cong\{L(\lambda) : \lambda\in\{\text{dominant weights}\}\}$$

$$\lambda=\sum_{i=1}^{g}\lambda_i w_i \qquad w_i\text{: fund. weights.}$$

std rep: $k^{2g}=L(w_1)$

$$L(w_i)=\ker\Big(\Lambda^i k^{2g}\xrightarrow{2}\Lambda^{i-2}k^{2g}\Big)$$

contraction w/ the sympl. form.

$$\dim L(w_i)=\binom{2g}{i}-\binom{2g}{i-2}$$

In general, $\dim L(\lambda)$ is given by the Weyl character formula. (WCF)

Maximal torus $T\cong(k^*)^g=\begin{bmatrix}q_1 & & & & & \\ & \ddots & & & & \\ & & q_g & & & \\ & & & q_1^{-1} & & \\ & & & & \ddots & \\ & & & & & q_g^{-1}\end{bmatrix}$

$$\subset Sp(2g,k)$$

An irrep of T is 1D; $(q_1, \dots q_g)$

$$\lambda \in \mathrm{Hom}(T, K^*) \cong \mathbb{Z}^g$$

$$\epsilon_i(q_1 \dots q_g) = q_i$$

Weights of $L(w_1) = \{\epsilon_1 \dots \epsilon_g, -\epsilon_1, \dots, -\epsilon_g\}$

highest wt.

$w_i = \epsilon_1 + \dots + \epsilon_i$ WCF gives all other weights

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On to char $= p$: $Sp(2g, \overline{\mathbb{F}}_p)$

alg. closure of $\mathbb{F}_p$

irreps on $\overline{\mathbb{F}}_p$ v.s.

Usually $L(w_1)$ is not irreducible.

<u>Thm</u> (Chevalley, 50's) Irreps are in bijection w/ dominant wts,

$$\lambda \longleftrightarrow L_p(\lambda) \qquad \lambda = \sum_{i=1}^{g} \lambda_i w_i$$

but $\dim L_p(\lambda)$ is unknown.

comment: really this is a computational complexity issue.

λ is "p-restricted" if $0 \le \lambda_i \le p-1$

Thm (Steinberg) If λ is p-restricted then $L_p(\lambda)$ remains simple when restricted to $Sp(2g, \mathbb{F}_p)$. Moreover all irreps of $Sp(2g, \mathbb{F}_p)$ arise in this way.

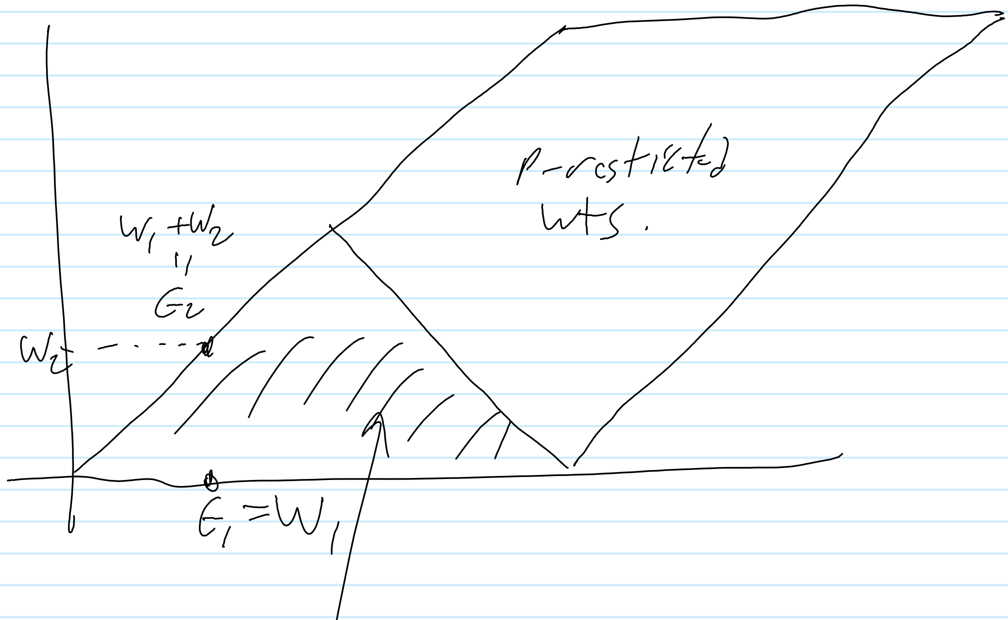
Thm (Gow, 1998) For $w_i \in \{w_g, \dots w_{g-p+2}\}$

$$L_p(w_i) = \frac{\ker(\Lambda^i \xrightarrow{\partial} \Lambda^{i-2})}{\operatorname{im}(\Lambda^{i+2(p-1)} \xrightarrow{\partial^{p-1}} \Lambda^i)}$$

Lustig Conjecture:

$g=2$

$p>2g$



For λ in fund. alcove $\dim L_p(\lambda)$ is still given by WCF.

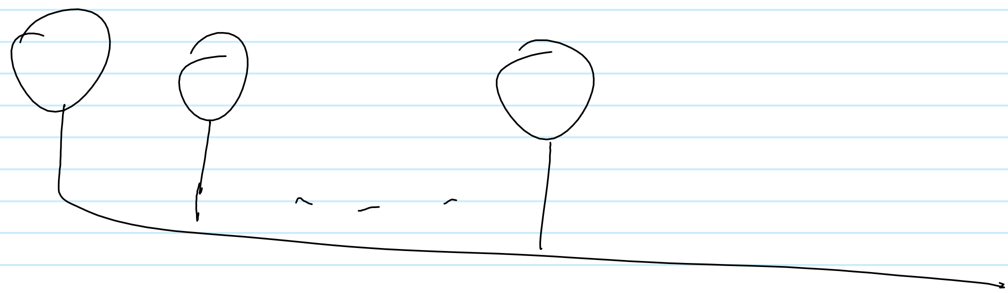
Lustig conj. is a formula for dims beyond the fund. alcove; known to be true for very large p.

Our (w/ Gilmer) irreps:

p: odd prime $\quad 0 \le c \le \frac{p-3}{2}$

$\epsilon \in \mathbb{Z}/2$

$F_g^\epsilon(p,c) = \mathbb{F}_p$-v.s. spanned by colourings of a graph G_g:

$G =$ 

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<u>Thm</u> $F_g^\epsilon(p,C)$ supports a structure of an irred $sp(2g, \mathbb{F}_p)$; these are a certain specified set of $L(\lambda)$'s.