

Lawrence on Explicit DGA Models of Simple Chain Complexes and their Properties

July-29-15 7:30 AM

Concepts: *

- * Lie algebras
- * diff. geom. concepts
- * Functoriality
- * Bernoulli numbers

A : connection

$$F_A = dA + A \wedge A$$

$$\int_{M/g} e^{\frac{i\kappa}{4\pi} CS(A)} \mathcal{D}A$$

$x \mapsto g^{-1} dx g + g^{-1} x g$ inf. gauge group action:

$$\delta x = dh - [h, x]$$

Goal: * Associate a DGLA to a cell complex in an "interestingly meaningful" manner

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Motivation:

① Quillen's rational homotopy theory.

③ Deformation Theory.

Graded Lie Algebra:

$$A = \bigoplus_{n \in \mathbb{Z}} A_n$$

$$[a, b] = -(-1)^{|a||b|} [b, a]$$

$$ad_{[a, b]} = [ad_a, ad_b]$$

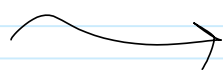
$$ad_{\partial a} = [\partial, ad_a] \quad \partial^2 = 0$$

Maurer-Cartan eqn':

$$\partial a = -\frac{1}{2} [a, a]$$

\Leftrightarrow] DGLA deformation $(A, \partial_a = \partial + ad_a)$

~~Cell complex~~



DGLA freely generated as a Lie alg. by gen. for each cell, w/ degree $k-1$.

conditions on ∂ :

* 0 cells $a \rightsquigarrow \partial a = -\frac{1}{2} [a, a]$

* ∂ is local:

$$\partial(c) \in \langle \bar{c} \rangle$$

* IF $\partial = \partial_0 + \partial_1 + \partial_2$ ∂_r of deg r in brackets

then ∂_0 is the topological bndy.

Kontsevich:

$$\partial e = \text{ad}_e b + \sum_{n=0}^{\infty} \frac{B_n}{n!} (\text{ad}_e)^n (b-a)$$

where
$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n x^n}{n!}$$

Joint work w/ Dennis Sullivan:

$e \in A_0$ defines a flow on A_{-1} by

$$\dot{u} = \partial e - \text{ad}_e u \quad u(t) \in A_{-1}$$

w/ $u(0) = a$, get

$$u(t) = \left(\frac{1 - e^{-t \text{ad}_e}}{\text{ad}_e} \right) (\partial e) + e^{-t \text{ad}_e} a$$

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