# Lawrence on Explicit DGA Models of Simple Chain Complexes and their Properties

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Concepts: * Lie algebras
* diff. geom. concepts
* Functoriality
* Bernoulli numbers

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$A$: connection

$F_A = dA + A\wedge A$

$$\int_{\mathcal{A}/\mathcal{G}} e^{\frac{ik}{4\pi}CS(A)}\mathcal{D}A$$

$x \mapsto g^{-1}dg + g^{-1}xg$ inf. gauge group action: $\delta x = dh - [h,x]$

Goal: * Associate a DGLA to a cell complex in an "interestingly meaningful" manner

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Motivation:

(A) Quillen's rational homotopy theory.

# (B) Deformation Theory.

Graded Lie Algebra: $A = \bigoplus_{n \in \mathbb{Z}} A_n$

$$[a,b] = -(-1)^{|a||b|}[b,a]$$

$$ad_{[a,b]} = [ad_a, ad_b]$$

$$ad_{\partial a} = [\partial, ad_a] \qquad \partial^2 = 0$$

Maurer-Cartan eqn':

$$\partial a = -\tfrac{1}{2}[a,a]$$

$\Longleftrightarrow$ $\exists$ DGLA deformation $(A, \partial_a = \partial + ad_a)$

$X$ cell complex $\rightsquigarrow$ DGLA freely gen as a Lie alg. by gen. for each cell, w/ degree $k-1$.

conditions on $\partial$:

* 0 cells $a \rightsquigarrow \partial a = -\frac{1}{2}[a,a]$
* $\partial$ is local:

$$\partial(c) \in \langle \bar{c} \rangle$$

* If $\partial = \partial_0 + \partial_1 + \partial_2$ $\quad$ $\partial_r$ of deg $r$ in brackets

then $\partial_0$ is the topological bndry.

Kontsevich:

$$\partial e = ad_e b + \sum_{n=0} \frac{B_n}{n!} (ad_e)^n (b-a)$$

where $$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n x^n}{n!}$$

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Joint work w/ Dennis Sullivan:

$e \in A_0$ defines a flow on $A_{-1}$ by

$$\dot{u} = de - ad_e u \qquad u(t) \in A_{-1}$$

w/ $u(0) = a$, get

$$u(t) = \left( \frac{1 - e^{-t\, ad_e}}{ad_e} \right) (\partial e) + e^{-t\, ad_e} a$$

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