Kashaev on Quantum Dilogarithm and Self-Dual LCA Groups

July-28-15 3:29 AM

Joint w/ J. Andersen.

Motivation: understand quantum CS theory with non-compact gauge group, especially GL(2)

---- generalized TQFT based on ∞-dim v.s.

$GL_2\mathbb{C}$ CS $>$ $GL_2\mathbb{R}$ CS $>$ Teichmüller space

quantization

quantum Teichmüller Theory.

Complex Teichmüller TQFT, $>$ Teichmüller TQFT $>$ Quantum Teich. on SLCAG (Self-dual-locally-compact Abelian groups)

$\vee$ Teich TQFT on SDLCAG $>$

Today.

Quantum Teichmüller

$\in S = \{(V, T, R) :$ V: cmplx v.s. $T \in \mathrm{End}(V \otimes V)$ $\}$

$$\in \mathcal{S} = \left\{ (V, T, R) : \begin{array}{l} V\text{: cmplx v.s.} \\ T \in \mathrm{End}(V \hat{\otimes} V) \\ R \in \mathrm{End}(V) \end{array} \right\}$$

s.t.

(T) $\quad T_{12} T_{13} T_{23} = T_{23} T_{12}$

(R) $\quad R^2 = R^{-1} \quad$ (Same as $R^3 = \mathrm{Id}$)

(TR) $\quad T R_1 P T = \zeta R_1 R_2$

P: permutation; ζ in $\mathbb{C}^*$

Example: $V = L^2(\mathbb{R})$

$$R = \alpha e^{2\pi i \hat{q}} e^{\pi i (\hat{p} + \hat{q})^2} \qquad \alpha \in \mathbb{T} = \{|z| = 1\}$$

$$T = e^{2\pi i \hat{p}_1 \hat{q}_2} \frac{1}{\Phi_b(\hat{q}_1 - \hat{q}_2 + \hat{p}_2)}$$

$$\Phi_b(x) = \exp\left(\frac{1}{4} \int_{\mathbb{R} + i\epsilon} \frac{e^{-2ixz}}{\sinh(bz)\sinh(b^{-1}z)} \frac{dz}{z} \right)$$

"Faddeev's quantum dilogarithm"

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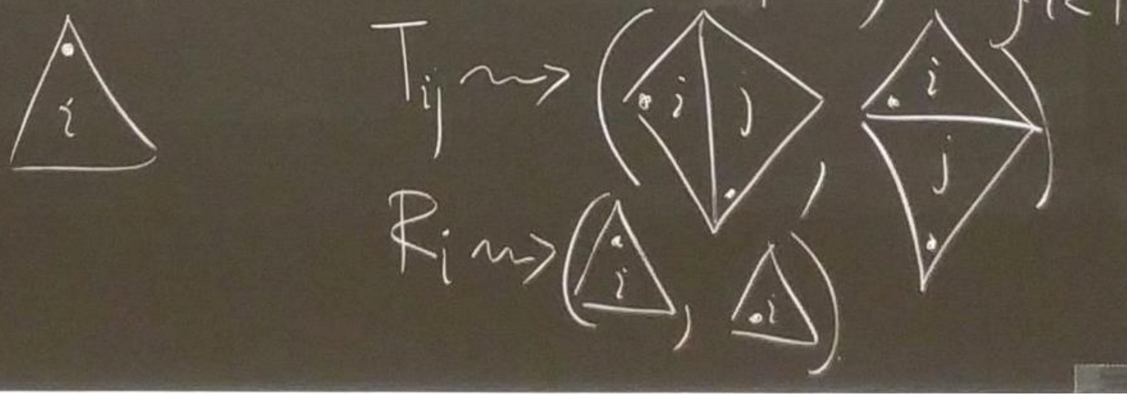
Any element of S realizes (decorated)
Ptolemy groupoid of ideal triangulations of a punctured surface Σ of negative $\chi(\Sigma)$.



For quantum Teichmüller:

$(T) \Longleftrightarrow \Phi_b(\hat{p})\Phi_b(\hat{q}) = \Phi_b(\hat{q})\Phi_b(\hat{p}+\hat{q})\Phi_b(\hat{p})$ (Faddeev 1994)

$(TR) \Longleftrightarrow \Phi_b(x)\Phi_b(-x) = \Phi_b(0)^2 e^{\pi i x^2}$

<u>Denote</u>: $g(x) = e^{\pi i x^2}$, $\chi(x,y) = \dfrac{g(x+y)}{g(x)g(y)} = e^{2\pi i x y}$.

Define unitary operators: F, G, Φ in $L^2(\mathbb{R})$

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