Justifying r_2

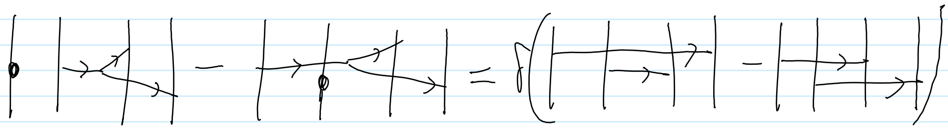
July-02-15 5:25 PM

$$[a_{jk}, a_{jl}] = c_l a_{jk} - c_k a_{jl} =: \gamma_{jkl},$$
$$[a_{jk}, a_{ik}] = b_i a_{jk} - b_j a_{ik},$$
$$[a_{jk}, a_{kl}] = b_j a_{kl} - b_k a_{jl} - \gamma_{jkl},$$
$$\operatorname{ad} a_{jk} : b_j, -b_k, -c_j, c_k \mapsto \delta a_{ij} - b_i c_j =: \gamma_{ij},$$

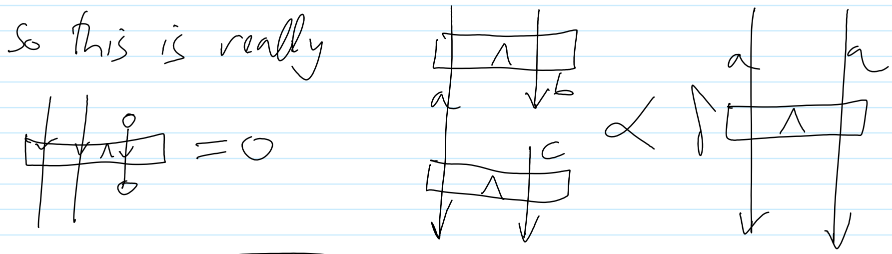
$$r_1 : \ b_i \gamma_{ijk} = \gamma_{ij} a_{ik} - \gamma_{ik} a_{ij}$$
$$r_2 : \ b_i \gamma_{jkl} = \gamma_{jk} a_{il} - \gamma_{jl} a_{ik}$$

$$r_2 : \ b_i \gamma_{jkl} = b_i c_l a_{jk} - b_i c_k a_{jl} = \delta a_{jk} a_{il} - b_j c_k a_{il} - \delta a_{jl} a_{ik} + b_j c_l a_{ik}$$

So
$$b_i \gamma_{jkl} - b_j \gamma_{ikl} = \delta (a_{jk} a_{il} - a_{jl} a_{ik})$$



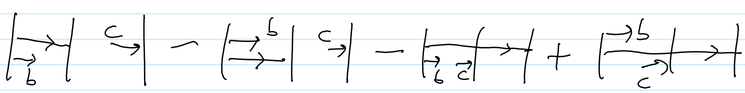
So this is really



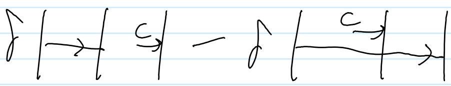
$$r_1 : \ b_i \gamma_{ijk} = \underbrace{\delta a_{ij} a_{ik} - \delta a_{ik} a_{ij}}_{0} \underbrace{- b_i c_j a_{ik} + b_i c_k a_{ij}}_{b_i \gamma_{ijk}}$$

r_2 implies

$\delta [a_{ij}, a_{ik}] =$



but is is also



Goal? Describe the minimal Lie-subalgebra of A^{v2D} that contains all the a_{ij}'s.