Habiro on Functors on Lagrangian Cobordisms Associated to Ribbon Hopf Algebras

July-30-15 5:02 AM

Joint with Le.

$\mathfrak{g}$: simple Lie algebra

$$\begin{tikzcd} \{\mathbb{Z}HS\} \arrow[r, "J = J^{\mathfrak{g}}"] \arrow[dr, bend right, "\text{RT invts}"'] & \widehat{\mathbb{Z}[q]} = \varprojlim \mathbb{Z}[q]/((q)_n) \arrow[d, "ev_\zeta"] \arrow[dl, phantom, "\circlearrowleft"] \\ & \mathbb{Z}[\zeta] \end{tikzcd}$$

$\zeta \in \sqrt{1}$

<u>Problem</u> Construct a TQFT version of $J^{\mathfrak{g}}$ unifying (part of) RT TQFTs.

Expect:

$$J^{\mathfrak{g}} : \mathcal{L}Cob \longrightarrow Mod_{\widehat{\mathbb{Z}[q]}}$$

$\uparrow$ Lagrangian cobordisms.

sl_2 is done, we expect all to work for higher $\mathfrak{g}$'s.

$\mathcal{L}Cob$

$\cap$

Cob

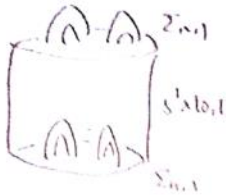
The category Cob (Crane-Yetter, Kerler)

$\mathrm{Ob}(\mathrm{Cob}) = \{0, 1, \cdots\}$

$\mathrm{Cob}(m,n) = \{\text{connected, oriented 3d cobordisms } \Sigma_{m,1} \to \Sigma_{n,1}\}/\cong$

$\Sigma_{m,1} =$ genus m

$M : m \to n$



$\partial M \cong -\Sigma_{m,1} \cup S^1\times[0,1] \cup \Sigma_{n,1}$

(Crane-Yetter, Kerler)
Fact (1) Cob is a braided monoidal category.

(2) $1 \in \mathrm{Ob}(\mathrm{Cob})$ admits a Hopf algebra (object) structure in Cob,

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Fact. $\mathrm{Aut}_{\mathrm{Cob}}(m) \cong \mathcal{M}_{m,1}$ the mapping class group of $\Sigma_{m,1}$

$\mathcal{B} \xrightarrow{J = J^{\mathfrak{g}}} \mathrm{Mod}_H$ H: any ribbon Hopf alg. over any commutative ring; E.g. $U_h(\mathfrak{g})$

$\cap$ problem: extend.

$\mathcal{L}\mathrm{Cob} \dashrightarrow \mathrm{Mod}_{\widehat{\mathbb{Z}[q]}}$ H: Hopf alg. w/ some props.

$\cap$

Cob

$\uparrow$

$\widetilde{\mathrm{Cob}} \xrightarrow{Z^{\mathrm{hen}}} \mathrm{Mod}_H^{\mathrm{f.d.}}$ H: factorizable f.d. ribbon Hopf alg. over a ...

$\widetilde{\mathcal{C}ob} \xrightarrow[\text{Kerler-Henning's TQFT}]{Z^{Hen}} \mathrm{Mod}_H^{f.d.}$

H: factorizable ribbon Hopf alg. over a field k.

Expectations:

① specializations to the Gilmer-Masbaum TQFT (integral version of RT)

$$\begin{tikzcd} \mathcal{L}\mathcal{C}ob \arrow[r, "J^g"] \arrow[d, hook] & \mathrm{Mod}_{\widehat{\mathbb{Z}[q]}} \arrow[d, "ev_\zeta"] \\ \mathcal{C}ob \arrow[r, "\text{G-M integral TQFT}"'] & \mathrm{Mod}_{\mathbb{Z}[\zeta]} \end{tikzcd}$$

② Injectivity on Lagrangian preserving MCG

$$\begin{array}{ll} \mathcal{L}g & \\ \cap & \\ \mathcal{L}\mathcal{C}ob & \mathrm{Aut} = \text{Lagrangian preserving MCG} \\ \cap & \\ \mathcal{C}ob & \mathrm{Aut} = \text{MCG's} \end{array}$$

$\rightsquigarrow$ should follow from asymp. faithfulness of RT TQFT of MCG (Andersen).
May also be a direct proof.

3 Relation to LMO Functor: The $\mathcal{L}Cob$ version of LMO by Cheptea-Massuyeau-Habiro.

[Århus $\int$ used]

$$\begin{tikzcd} P\mathcal{L}Cob \arrow[r, "Z^{LMO}"] \arrow[d, "J^{\mathfrak{g}}"'] \arrow[dr, phantom, "\text{up to nat. iso.}"] & \mathcal{A} \arrow[d, "W_{\mathfrak{g}}"] \\ Mod_{\mathbb{Z}[q]} \arrow[r, "q \mapsto e^h"'] & Mod_{\mathbb{Q}[[h]]} \end{tikzcd}$$

↑ parenthesized version.

$\mathcal{A}$: a category of diagrams

obj: integers

morphisms: Jacobi diagrams w/ no top struts.

Further in HabiroBlackboards.pdf.