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M — compact manifold.

Def M is said to be rigid, if whenever a manifold is homotopy equiv. to M, it is homeomorphic to M.

Examples Σ^2 are rigid.
S^3 is rigid by Poincaré.
(some) Lens spaces are not rigid

Def M is "aspherical" if $\tilde{M}$, it's universal cover, is contractible.

(stable) Borel conj: All aspherical manifolds are rigid.

M is "stably rigid" if whenever N is hom. equiv to N, $N \times \mathbb{R}^n \cong M \times \mathbb{R}^n$ for some n.

Novikov's Thm: The Pontryagin classes are topological invariants.

Borel + Novikov $\Rightarrow$ Rational Pont. classes are homotopy invariants.

This is "an infinitesimal Borel Conj", equiv. to the Novikov Conj. for aspherical mflds.

<u>Thm</u> If a fundamental group is coarsely embeddable in a Hilbert space, the Novikov conj. holds for G.

Coarsly embeddable in H: $\exists f: G \to H$ s.t.

1. $\forall$ finite $F \subset G$ $\exists r \geq 0$ s.t.

$$gh^{-1} \in F \Rightarrow \|f(g)-f(h)\| \leq r$$

2. $\forall R > 0$ $\exists$ finite $E \subset G$ s.t.

$$g^{-1}h \notin E \Rightarrow \|f(g)-f(h)\| \geq R$$

<u>Thm</u> (G, ...) If G has finite decomposition complexity, then the stable Borel conjecture holds.

"Finite decomposition complexity" will not be defined here, yet this condition implies coarse embeddability and includes all interesting examples.

Algebraic Novikov Conjecture for

$$S = \bigcup_{p=1}^{\infty} S_p \qquad S_p = \{T \in B(H) : tr(?) < \infty\}$$

⋮

Some K-theoretic analog holds over $\mathbb{Q}$.