Dunfield on A Tale of Two Norms

July-27-15 7:30 AM

Joint w/ Brock, partially also w/ Hirani.

M^3: Closed oriented hyperbolic 3-mnfld.

$\phi \in H^1(M;\mathbb{Z}) \cong H_2(M;\mathbb{Z})$

we measure "complexity" of ϕ.

Topology $\|\ \|_{Th}$	Geometry $\|\ \|_{L^2}$
minimal genus of a surface representing ϕ in H_2 "Thurston norm"	Harmonic norm: L^2-norm on harmonic representatives in H^1.

[Bergeron–Şengün–Venkatesh 2014] The two are related.

Conjecture (Bergeron–Venkatesh, Le)

$M = M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow M_3$

with $\operatorname{inj} M_n \to \infty$ then in nice circumstances

$H_1(M_n,\mathbb{Z})_{\text{torsion}}$ grows exponentially fast:

$\lim \dfrac{\log |H_1(M_n,\mathbb{Z})\ \text{torsion}|}{\ }$ |

$$\lim_{n\to\infty} \frac{\log |H_1(M_n,\mathbb{Z}) \text{ torsion}|}{\mathrm{Vol}(M_n)} = \frac{1}{6\pi}$$

($\le$ proven by Le 2015)

[Brock – D 2015] For all closed hyp M^3,

$$\frac{\pi}{\sqrt{\mathrm{Vol}(M)}} \|\cdot\|_{Th} \le \|\cdot\|_{L^2} \le 4\epsilon^{-1/2} \|\cdot\|_{Th}$$

where $\epsilon = \min(\mathrm{inj}\, M, 0.14)$.

Least area norm: $\phi \in H^1(M,\mathbb{Z})$

$$\|\phi\|_{LA} \underset{\text{realized}}{=} \inf\{\mathrm{Area}(S) : S \text{ is dual to } \phi\}$$

$$\underset{\text{by geom. measure theory}}{=} \|\phi\|_{L^1} = \inf\{\|\alpha\|_{L^1} : \alpha \text{ represents } \phi\}$$

<u>Lemma</u> $\pi\|\cdot\|_{Th} \underset{\substack{\text{not easy,}\\ \text{Schoen–Uhlenbeck}}}{\le} \|\cdot\|_{LA} \underset{\text{easy}}{\le} 2\pi\|\cdot\|_{Th}$

0:54