Deriving Γ-calculus

July-12-15 12:12 AM

Deriving Gassner. $\mathcal{L}^{2Dw}$ is $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo locality, $[a_{ij}, a_{ik}] = 0$, $[a_{ik}, a_{jk}] = -[a_{ij}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, and $(\text{mod } \langle a_{ii}\rangle)$ $[a_{ij}, a_{ji}] = b_i a_{ji} - b_j a_{ij}$. Acts on $V = \mathbb{Q}[\![b_i]\!]\langle x_i = a_{i\infty}\rangle$ by $[a_{ij}, x_i] = 0$, $[a_{ij}, x_j] = b_i x_j - b_j x_i$. Hence $e^{\operatorname{ad} a_{ij}} x_i = x_i$, $e^{\operatorname{ad} a_{ij}} x_j = e^{b_i} x_j + \frac{b_j}{b_i}(1 - e^{b_i}) x_i$. Renaming $\bar{x}_i = x_i / b_i$, $t_i = e^{b_i}$, get $[e^{\operatorname{ad} a_{ij}}]_{\bar{x}_i, \bar{x}_j} = \begin{pmatrix} 1 & 1 - t_i \\ 0 & t_i \end{pmatrix}$.

The $\mathcal{L}^{2Dw}$ Adjoint representation. $e^{\operatorname{ad} a_{ij}}$ acts by

$$a_{kl} \mapsto a_{kl}, \qquad a_{ik} \mapsto a_{ik}, \qquad a_{kj} \mapsto e^{-b_i} a_{kj} + \frac{b_k}{b_i}(1 - e^{-b_i}) a_{ij},$$

$$a_{ki} \mapsto a_{ki} + (1 - e^{-b_i}) a_{kj} + b_k \frac{e^{-b_i} - 1}{b_i} a_{ij},$$

$$a_{jk} \mapsto e^{b_i} a_{jk} + \frac{b_j}{b_i}(1 - e^{b_i}) a_{ik}, \quad a_{ji} \mapsto e^{b_i} a_{ji} + \frac{b_j}{b_i}(1 - e^{b_i}) a_{ij}.$$

irrelevant for stitching!

The stitching operation seems independent of the specifics! Why not use it elsewhere? Does it have an abstract meaning?

Does stitching make sense on $\text{End}(V^{\otimes n})$?

Yes, it is a V^*-V contraction. It is linear.

Does it make sense on arbitrary spaces associated with V_n?