Pensieve header: Ad hoc unitarity for one-up one-down pure tangles.

```
<< KnotTheory`
```

```
Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
SetDirectory["C:/drorbn/AcademicPensieve/2015-07/"];
Get["../Projects/MetaCalculi/MetaCalculi-Program.m"]
```

$$\Omega a = \begin{pmatrix} \frac{1}{1-T_1} & 0 \\ 1 & \frac{1}{1-T_2} \end{pmatrix};\ \Omega b = \begin{pmatrix} \frac{1}{1-T_1} & 1 \\ 0 & \frac{1}{1-T_2} \end{pmatrix};$$

```
{L = Knot[10, 12], d = 2};
n = Crossings[L];
λ = Times @@ PD[L] /.
    X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]], Xp[l, i], Xm[j, i]];
ξ = Γ[Drop[λ, {d}]];
{min, max} = Sort[List @@ λ[[d]]];
Do[ξ = dm[min + 1, i, min + 1][ξ], {i, min + 2, max - 1}];
If[max < 2 n,
  Do[ξ = dm[max + 1, i, max + 1][ξ], {i, max + 2, 2 n}];
  Do[ξ = dm[max + 1, i, max + 1][ξ], {i, 1, min - 1}];
  ξ = dσ[min + 1 → 1, max + 1 → 2][ξ],
  (*else*) Do[ξ = dm[1, i, 1][ξ], {i, 2, min - 1}];
  ξ = dσ[min + 1 → 2][ξ]
 ];
```

```
λ
```

```
Xm[2, 9] Xm[4, 1] Xm[10, 3] Xp[6, 15] Xp[8, 17]
 Xp[12, 19] Xp[14, 5] Xp[16, 7] Xp[18, 13] Xp[20, 11]
```

```
U = Simplify[ξ@A]
```

$$\left\{\left\{\frac{T_2\left(-1+T_2-T_2^2+T_2^3\right)+T_1\left(2-2T_2+2T_2^2-2T_2^3+T_2^4\right)}{T_2\left(-1+T_2-T_2^2+T_2^3+T_1\left(3-4T_2+4T_2^2-4T_2^3+2T_2^4\right)\right)},\right.\right.$$
$$\left.\frac{(-1+T_1)\left(2-3T_2+3T_2^2-3T_2^3+2T_2^4\right)}{T_2\left(-1+T_2-T_2^2+T_2^3+T_1\left(3-4T_2+4T_2^2-4T_2^3+2T_2^4\right)\right)}\right\},$$
$$\left\{\frac{T_1(-1+T_2)\left(2-3T_2+3T_2^2-3T_2^3+2T_2^4\right)}{T_2\left(-1+T_2-T_2^2+T_2^3+T_1\left(3-4T_2+4T_2^2-4T_2^3+2T_2^4\right)\right)},\right.$$
$$\left(2-4T_2+4T_2^2-4T_2^3+3T_2^4+T_1\left(-2+6T_2-7T_2^2+7T_2^3-6T_2^4+2T_2^5\right)\right)/$$
$$\left.\left.\left(T_2\left(-1+T_2-T_2^2+T_2^3+T_1\left(3-4T_2+4T_2^2-4T_2^3+2T_2^4\right)\right)\right)\right\}\right\}$$

```
{Eigenvectors[U], Eigenvalues[U],
 Simplify[# (# /. T_i_ :> 1/T_i)] & /@ Eigenvalues[U]}
```

$$\left\{\left\{\{-1,1\},\left\{-\frac{1-T_1}{T_1(-1+T_2)},1\right\}\right\},\left\{\left(2-4T_2+T_1T_2+4T_2^2-T_1T_2^2-4T_2^3+T_1T_2^3+3T_2^4-T_1T_2^4\right)/\right.\right.$$
$$\left.\left.\left(T_2\left(-1+3T_1+T_2-4T_1T_2-T_2^2+4T_1T_2^2+T_2^3-4T_1T_2^3+2T_1T_2^4\right)\right),1\right\},\{1,1\}\right\}$$

```
Ωa.Inverse[U] // Simplify // MatrixForm
```

$$\begin{pmatrix} \frac{2-4 T_2+4 T_2^2-4 T_2^3+3 T_2^4+T_1 (-2+6 T_2-7 T_2^2+7 T_2^3-6 T_2^4+2 T_2^5)}{(-1+T_1)(-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4)} & \frac{-2+3 T_2-3 T_2^2+3 T_2^3-2 T_2^4}{-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4} \\ \frac{-2+(4-3 T_1) T_2+4(-1+T_1) T_2^2-4(-1+T_1) T_2^3+(-3+4 T_1) T_2^4-2 T_1 T_2^5}{-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4} & \frac{2+3(-2+T_1) T_2+(7-4 T_1) T_2^2+(-7+4 T_1) T_2^3+(6-4 T_1) T_2^4+2(-1+T_1)}{(-1+T_2)(-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4)} \end{pmatrix}$$

```
Transpose[U /. {T_i_ :> 1/T_i}].Ωa // Simplify // MatrixForm
```

$$\begin{pmatrix} \frac{2-4 T_2+4 T_2^2-4 T_2^3+3 T_2^4+T_1 (-2+6 T_2-7 T_2^2+7 T_2^3-6 T_2^4+2 T_2^5)}{(-1+T_1)(-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4)} & \frac{-2+3 T_2-3 T_2^2+3 T_2^3-2 T_2^4}{-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4} \\ \frac{-2+(4-3 T_1) T_2+4(-1+T_1) T_2^2-4(-1+T_1) T_2^3+(-3+4 T_1) T_2^4-2 T_1 T_2^5}{-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4} & \frac{2+3(-2+T_1) T_2+(7-4 T_1) T_2^2+(-7+4 T_1) T_2^3+(6-4 T_1) T_2^4+2(-1+T_1)}{(-1+T_2)(-2-(-4+T_1) T_2+(-4+T_1) T_2^2-(-4+T_1) T_2^3+(-3+T_1) T_2^4)} \end{pmatrix}$$

```
TestUnitarity[L_, d_] := Module[{n, λ, ξ, min, max},
  n = Crossings[L];
  λ = Times @@ PD[L] /.
    X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]], Xp[l, i], Xm[j, i]];
  ξ = Γ[Drop[λ, {d}]];
  {min, max} = Sort[List @@ λ[[d]]];
  Do[ξ = dm[min + 1, i, min + 1][ξ], {i, min + 2, max - 1}];
  If[max < 2 n,
   Do[ξ = dm[max + 1, i, max + 1][ξ], {i, max + 2, 2 n}];
   Do[ξ = dm[max + 1, i, max + 1][ξ], {i, 1, min - 1}];
   ξ = dσ[min + 1 → 1, max + 1 → 2][ξ],
   (*else*) Do[ξ = dm[1, i, 1][ξ], {i, 2, min - 1}];
   ξ = dσ[min + 1 → 2][ξ]
  ];
  U = Simplify[ξ@A];
  Simplify[(Ωa == Transpose[U /. {T_i_ :> 1/T_i}].Ωa.U) ⋁
    (Ωb == Transpose[U /. {T_i_ :> 1/T_i}].Ωb.U)]
 ]
```

```
TestUnitarity[L, 1]
```

```
True
```

```
Table[
 Table[TestUnitarity[L, d], {d, Crossings[L]}],
 {L, AllKnots[{3, 8}]}
]
```

```
{{True, True, True}, {True, True, True, True},
 {True, True, True, True, True}, {True, True, True, True, True},
 {True, True, True, True, True, True}, {True, True, True, True, True, True},
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 {True, True, True, True, True, True, True, True},
 {True, True, True, True, True, True, True, True},
 {True, True, True, True, True, True, True, True}}
```

$$\left(U = \text{Simplify}\left[\begin{pmatrix} 1/\alpha & \beta/\alpha \\ -\gamma/\alpha & (\alpha\delta - \gamma\beta)/\alpha \end{pmatrix} /. \{\alpha \to 1-\gamma,\ \delta \to 1-\beta\}\right]\right) \text{ // MatrixForm}$$

$$\begin{pmatrix} \frac{1}{1-\gamma} & \frac{\beta}{1-\gamma} \\ \frac{\gamma}{-1+\gamma} & \frac{-1+\beta+\gamma}{-1+\gamma} \end{pmatrix}$$

$$\left(U = \text{Simplify}\left[\begin{pmatrix} 1-\gamma & \beta \\ \gamma & 1-\beta \end{pmatrix}\right]\right) \text{ // MatrixForm}$$

$$\begin{pmatrix} 1-\gamma & \beta \\ \gamma & 1-\beta \end{pmatrix}$$

Factor$\left[(-1+T_1)\,(-1+T_2)\,(\Omega a - \text{Transpose}[U\ /.\ \{\beta\to\beta_c,\ \gamma\to\gamma_c\}].\Omega a.U)\right]$ **// MatrixForm**

$$\begin{pmatrix} \gamma - \gamma T_2 - \gamma \gamma_c + T_1 \gamma_c - T_1 T_2 \gamma_c + \gamma T_1 T_2 \gamma_c & -\beta + \beta T_2 - \gamma_c + \beta \gamma_c + T_1 \gamma_c - \beta T_1 T_2 \gamma_c \\ -\gamma T_2 + \gamma T_1 T_2 + \gamma \beta_c - T_1 \beta_c + T_1 T_2 \beta_c - \gamma T_1 T_2 \beta_c & \beta T_2 - \beta T_1 T_2 + \beta_c - \beta \beta_c - T_1 \beta_c + \beta T_1 T_2 \beta_c \end{pmatrix}$$

Eigenvalues$\left[\begin{pmatrix} 1-\gamma & \beta \\ \gamma & 1-\beta \end{pmatrix}\right]$

$\{1,\ 1-\beta-\gamma\}$

Eigenvalues$\left[\begin{pmatrix} \frac{1}{1-\gamma} & \frac{\beta}{1-\gamma} \\ \frac{\gamma}{-1+\gamma} & \frac{-1+\beta+\gamma}{-1+\gamma} \end{pmatrix}\right]$

$\left\{\frac{-1+\beta}{-1+\gamma},\ 1\right\}$