Pensieve header: One-Co computations scavenging base for abc.nb.

The bracket

Generalities

```
CF[expr_] := expr /. {
   a[f_, i_, j_] :> Interpretation[StringForm["a[``,````]", f, i, j], a[f, i, j]],
   γ[f_, i_, j_] :> Interpretation[StringForm["γ[``,````]", f, i, j], γ[f, i, j]],
   γ[f_, i_, j_, k_] :> Interpretation[StringForm["γ[``,``````]", f, i, j, k], γ[f, i, j, k]],
   γa[f_, i_, j_, k_, l_] :>
    Interpretation[StringForm["γa[``,````````]", f, i, j, k, l], γa[f, i, j, k, l]]
  }
```

Specific Brackets

```
tests = {β[f[bj, bk]], γ[1, i, j], γ[1, i, k], γ[1, i, l], γ[1, j, k], γ[1, j, l], γ[1, k, l],
   γ[1, j, l, m], γ[1, i, j, l], γ[1, i, k, l], γ[1, k, l, m], γ[1, j, k, l], γ[1, i, j, k],
   a[1, j, k], a[1, j, l], a[1, i, k], a[1, i, j], a[1, k, l], a[1, l, m], γa[1, j, k, j, l]};
shows = Complement[tests, {γ[1, i, l], γ[1, j, l], γ[1, j, l, m], a[1, j, k], a[1, l, m]}];
```

OneCoBrackets

```
ColumnForm[(# → B[a[1, j, k], #]) & /@ shows] // CF
```

OneCoBrackets

$$a[1,ij] \to a[-b_i,jk] + a[b_j,ik] + \gamma[1,ijk]$$
$$a[1,ik] \to a[b_i,jk] + a[-b_j,ik]$$
$$a[1,jl] \to \gamma[1,jkl]$$
$$a[1,kl] \to a[b_j,kl] + a[-b_k,jl] + \gamma[-1,jkl]$$
$$\beta[f[b_j, b_k]] \to \gamma[-f^{(0,1)}[b_j, b_k] + f^{(1,0)}[b_j, b_k], jk]$$
$$\gamma[1,ij] \to \gamma[b_j,ik]$$
$$\gamma[1,ik] \to \gamma[-b_j,ik]$$
$$\gamma[1,jk] \to \gamma[-b_j,jk]$$
$$\gamma[1,kl] \to \gamma[b_j,kl] + \gamma[-b_k,jl]$$
$$\gamma[1,ijk] \to \gamma[-b_j,ijk] + \gamma a[1,ijjk] + \gamma a[1,ikjk]$$
$$\gamma[1,ijl] \to \gamma a[1,ikjl]$$
$$\gamma[1,ikl] \to \gamma a[-1,ikjl]$$
$$\gamma[1,jkl] \to \gamma a[-1,jkjl]$$
$$\gamma[1,klm] \to \gamma[b_j,klm] + \gamma[-b_k,jlm]$$
$$\gamma a[1,jkjl] \to \gamma a[-b_j,jkjl]$$

Testing Jacobi and Anti-Symmetry

```
DeleteCases[
 Flatten[Outer[
   Jacobi,
   FormalPlusBasis[4, f],
   FormalPlusBasis[4, g],
   FormalPlusBasis[4, h]
  ]],
 0]
```

The Adjoint action

```
Ad[a[t_, j_, k_]][a[1, j_, k_]] /; FreeQ[t, b_] := a[1, j, k];

Ad[a[t_, j_, k_]][a[1, j_, l_]] /; DQ[j, k, l] ∧ FreeQ[t, b_] :=
 a[1, j, l] + γ[t, j, k, l] + γa[(1 - e^(-t b_j) - t b_j)/b_j^2, j, k, j, l];

Ad[a[t_, j_, k_]][a[1, i_, k_]] /; DQ[i, j, k] ∧ FreeQ[t, b_] := a[e^(-t b_j), i, k] + a[((1 - e^(-t b_j)) b_i)/b_j, j, k] +
   γa[(e^(-2 t b_j) b_i (2 e^(t b_j) t b_j + 1 - e^(2 t b_j)))/b_j^3, j, k, j, k] + γa[(e^(-2 t b_j) (e^(t b_j) (1 - t b_j) - 1))/b_j^2, j, k, i, k];

Ad[a[t_, j_, k_]][a[1, i_, j_]] /; DQ[i, j, k] ∧ FreeQ[t, b_] := a[1, i, j] + a[1 - e^(-t b_j), i, k] +
   a[((e^(-t b_j) - 1) b_i)/b_j, j, k] + γ[(1 - e^(-t b_j))/b_j, i, j, k] + γa[(b_i (1 - 2 e^(-t b_j) t b_j - e^(-2 t b_j)))/b_j^3, j, k, j, k] +
   γa[(e^(-t b_j) + t b_j - 1)/b_j^2, i, j, j, k] + γa[(e^(-t b_j) + t b_j - 1)/b_j^2, i, k, j, k] + γa[(e^(-2 t b_j) (1 + e^(t b_j) (-1 + t b_j)))/b_j^2, j, k, i, k];

Ad[a[t_, j_, k_]][a[1, k_, l_]] /; DQ[j, k, l] ∧ FreeQ[t, b_] :=
   a[e^(t b_j), k, l] + a[((1 - e^(t b_j)) b_k)/b_j, j, l] + γ[(t b_j b_k + (1 - e^(t b_j)) (b_j + b_k))/b_j^2, j, k, l] +
   γa[(1 + e^(t b_j) (t b_j - 1))/b_j^2, j, k, k, l] + γa[(e^(-t b_j) b_j + e^(t b_j) (b_j + 2 b_k - t b_j b_k) - 2 b_j - 2 b_k - t b_j b_k)/b_j^3, j, k, j, l];

Ad[a[t_, j_, k_]][a[1, l_, m_]] /; DQ[j, k, l, m] ∧ FreeQ[t, b_] := a[1, l, m];

Ad[x_][y_] := (Print["Ad not yet defined on ", {x, y}]; y);
```

OneCoAd

```
Print[CF[# -> Ad[a[t, j, k]][#]] & /@ shows;
```

Verifying R3

```
VerifyR3 /@ {a[f[b1, b2, b3, b4], 1, 4], a[f[b1, b2, b3, b4], 2, 4], a[f[b1, b2, b3, b4], 3, 4]}
```

```
{{True}, {True}, {γ[-(1/b2) e^b1 (-1 + e^b2) f[b1, b2, b3, b4] b3 (f0'[b2] - f0'[b3] + b1 (f1'[b2] - f1'[b3])), 2, 4] +
   γ[-(1/b1) f[b1, b2, b3, b4] b3 ((-1 + e^b1) f1[b2] + e^b1 (-1 + e^b2) g[b1, b2] b1 + e^b1 g[b1, b3] b1 - e^(b1+b2) g[b1, b3] b1 -
       e^b2 g[b2, b3] b2 + e^(b1+b2) g[b2, b3] b2 - e^b2 f0'[b3] + e^(b1+b2) f0'[b3] - e^b2 b2 f1'[b3] + e^(b1+b2) b2 f1'[b3]), 1, 4] +
   γ[((-1 + e^b1) f[b1, b2, b3, b4] (2 - 2 e^b2 + (1 + e^b2) b2) b3)/(2 b1 b2), 1, 2, 4] + γ[(1/(2 b1^4 b2)) e^-b1 (-1 + e^b1)
     f[b1, b2, b3, b4] (2 e^b2 (-1 + e^b1) b2^3 - e^b2 (1 + e^b1) b1 b2^3 + e^b1 b1^3 (2 - 2 e^b2 + (1 + e^b2) b2)) b3, 1, 3, 4] +
   γa[(1/(2 b1^4 b2)) e^-b1 (-1 + e^b1) f[b1, b2, b3, b4] (2 e^b2 (-1 + e^b1) b2^3 - e^b2 (1 + e^b1) b1 b2^3 + e^b1 b1^3 (2 - 2 e^b2 + (1 + e^b2) b2)),
    1, 3, 3, 4] + γa[-((-1 + e^b1) f[b1, b2, b3, b4] (2 - 2 e^b2 + (1 + e^b2) b2) b3)/(2 b1 b2^2), 1, 2, 2, 4] + γa[
   -(1/(b1^4 b2^2)) e^-b1 (-1 + e^b1) f[b1, b2, b3, b4] (2 e^b2 (-1 + e^b1) b2^3 - e^b2 (1 + e^b1) b1 b2^3 + e^b1 b1^3 (2 - 2 e^b2 + (1 + e^b2) b2)) b3,
   1, 3, 2, 4] == 0}}
```