

A Polynomial Time Knot Polynomial

Deriving Gassner. \mathcal{L}^{2Dw} is $\mathbb{Q}\langle b_i \rangle \langle a_{ij} \rangle$ modulo locality, $[a_{ij}, a_{ik}] = 0$, $[a_{ik}, a_{jk}] = -[a_{ij}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, and $(\text{mod } \langle a_{ii} \rangle) [a_{ij}, a_{ji}] = b_i a_{ji} - b_j a_{ij}$. Acts on $\mathbf{V} = \mathbb{Q}\langle b_i \rangle \langle \mathbf{x}_i = a_{i\infty} \rangle$ by $[a_{ij}, x_i] = 0$, $[a_{ij}, x_j] = b_i x_j - b_j x_i$. Hence $e^{\text{ad } a_{ij}} x_i = x_i$,

$e^{\text{ad } a_{ij}} x_j = e^{b_i} x_j + \frac{b_j}{b_i} (1 - e^{b_i}) x_i$. Renaming $y_i = x_i / b_i$, $t_i = e^{b_i}$, get

$$[e^{\text{ad } a_{ij}}]_{y_i, y_j} = \begin{pmatrix} 1 & 1 - t_i \\ 0 & t_i \end{pmatrix}.$$