What's Adjoint-Gassner?

June-17-15 5:53 PM

Adjoint Gassner. Renaming $\bar{a}_{ij} = a_{ij}/b_i$ and $t_i = e^{b_i}$, get

$$\bar{a}_{kj} \mapsto t_i^{-1}\bar{a}_{kj} + (1-t_i^{-1})\bar{a}_{ij},$$

$$\bar{a}_{ki} \mapsto \bar{a}_{ki} + (1-t_i^{-1})\bar{a}_{kj} + (t_i^{-1}-1)\bar{a}_{ij}$$

$$\bar{a}_{jk} \mapsto t_i\bar{a}_{jk} + (1-t_i)\bar{a}_{ik}, \quad \bar{a}_{ji} \mapsto t_i\bar{a}_{ji} + (1-t_i)\bar{a}_{ij}.$$

Gassner:

$$[e^{\mathrm{ad}\, a_{ij}}]_{\bar{x}_i,\bar{x}_j} = \begin{pmatrix} 1 & 1-t_i \\ 0 & t_i \end{pmatrix}.$$

$$a_{ki} \longrightarrow \begin{array}{c|ccc} & i & j & k \\ \hline i & & t_i^{-1}-1 & \\ j & & & \\ k & 1 & 1-t_i^{-1} & \end{array}$$

$$\begin{pmatrix} 1 & 1-t_i \\ 0 & t_i \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1-t_i \\ 0 & t_i \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & v \\ 0 & \\ 0 & \\ 0 & A \\ 0 & \\ 0 & \\ 0 & \end{pmatrix}$$

$$\begin{pmatrix} 1 & d \\ 0 & c \end{pmatrix}$$

$$\begin{pmatrix} 1 & b \\ 0 & a \end{pmatrix} \begin{matrix} 1 & d+bc \\ 0 & ac \end{matrix}$$

$$(ax+b)//(cx+d) = c(ax+b)+d$$

What are the n^2-dim reps of $M_{(n-1)\times(n-1)} \ltimes K^{n-1}$?

First ask "What's $\underset{\text{l.c.}}{\text{adjoint}}$ Gassner".

$[a_{ij}, a_{j\infty}] = -[a_{i\infty}, a_{j\infty}]$ so

$$(\mathrm{ad}\, a_{ij})(x_k) = \begin{cases} 0 & k \neq j \\ b_i x_j - b_j x_i & k = j \end{cases}$$

$$(\mathrm{ad}\, a_{ij})(\overline{x_j}) = b_i \overline{x_j} - b_i \overline{x_i} = b_i(\overline{x_j} - \overline{x_i})$$

so

$$(\mathrm{ad}\, \overline{a_{ij}})(\overline{x_j}) = \overline{x_j} - \overline{x_i}$$