Palmer: Homological stability for configuration spaces on closed manifolds

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M: manifold, smooth, paracompact, connected.

Def

ordered: $\tilde{C}_n(M) = \{x : \underline{n} \hookrightarrow M\}$

$C_n(M) = \tilde{C}_n(M)/S_n$

Why? * Braid groups

$M = \mathbb{R}^2$ $\pi_1(C_n(\mathbb{R}^2)) \cong \beta_n = \overset{\text{Artin}}{A_{n-1}}$

$\mathbb{R}^2 - pt$ B_n

$\mathbb{R}^2 - 2pt$ $\tilde{C}_n$

* Mapping spaces

$\Omega^n \Sigma^n X \simeq \coprod_{k \geq 0} C_k(\mathbb{R}^n; X) / \sim$?

* Convex geometry

K convex $\subseteq \mathbb{R}^2$, $n \geq 2$

Q $\exists$? partition $\underbrace{K_1 \dots K_n}_{\text{convex}}$ of K

w/ same area & same perimeter?

Reduced to a Q about the topology of $\tilde{C}_n(\mathbb{R}^2)$ by Kantorovic '39, proven for $n = p^l$ by Blagojevic-Ziegler '14

Homological stability.

$$X_1 \to X_2 \to \cdots \quad \cdots \to X_\infty$$

is "homologically stable" if

$$\forall i \geq 0 \;\; \exists N \geq 0 \;\; \forall n \geq N \quad H_i(X_n) \xrightarrow{\sim} H_i(X_{n+1})$$

$N = f(n)$

Examples

X_n	$f(n)$		X_∞ (w/ to homology)	
$B\Sigma_n$	$n/2$	Nakaoka	$\Omega^\infty_0 S^\infty$	Barratt-Priddy-Quillen
Braid groups → $B\beta_n$	$n/2$	Arnold	$\Omega^2_0 S^2$	(Segal)
$BGL_n(R)$	$\frac{n-1}{4}$	Charney	$K(R)$	(Quillen)
$BM(S_{g,1})$	$\frac{2}{3}g$	Harer	$\Omega^\infty_0 MTSO(2)$	(Madsen-Weiss)
$BAut(F_n)$	$\frac{n-2}{2}$	Hatcher-Vogtmann	$\Omega^\infty_0 S^\infty$	(Galatius)
$C_n(M)$	?	(here).		

$$C_n(M) \longrightarrow C_{n+1}(M)$$

(M non-compact, $M = \mathrm{int}(\overline{M})$, $\partial\overline{M} \neq \emptyset$)

by adding a pt near ∂M

(not always well-defined)

Thm (McDuff '75)

This induces isom's on $H_i(-,\mathbb{Z})$
for $i \le n/2$.

* Also for compact M,

$\exists : C_n(M) \xrightarrow{\text{scanning}} \Gamma_c(\dot{T}M)_n$

fibrewise 1-pt compactification

degree

compactly supported

inducing iso's in some range.

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