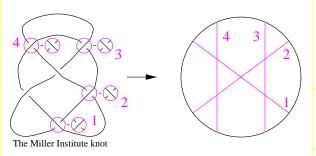
# Knotted Trivalent Graphs, Tetrahedra and Associators

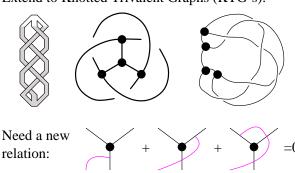
 $\omega := \text{http://www.math.toronto.edu/~drorbn/Talks/Louvain-1506}$ 

Handout, video, and links at  $\omega$ /

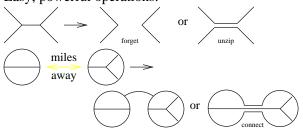
Goal: Z:{knots}->{chord diagrams}/4T so that



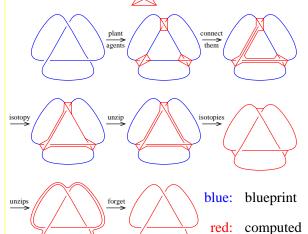
## Extend to Knotted Trivalent Graphs (KTG's):



#### Easy, powerful operations:

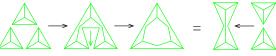


Using operations, KTG is generated by ribbon twists and the tetrahedron  $\uparrow$ :



Modulo the relation(s):



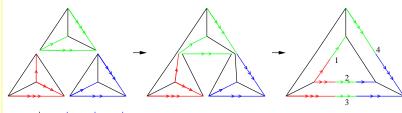


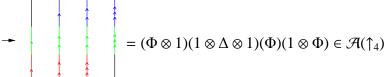
(+more)

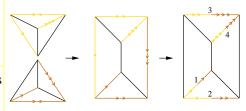
Claim. With  $\Phi := Z(\triangle)$ , the above relation becomes equivalent to the Drinfel'd's pentagon of the theory of quasi Hopf algebras.

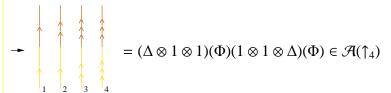
#### Proof.











### Ribbon Knots and Algebraic Knot Theory.

