OneCo-150520

May-20-15 9:45 AM

Cheat Sheet OneCo

http://drorbn.net/AcademicPensieve/2015-05/
initiated 14/4/15; modified 20/5/15, 7:30am; continues 2015-04

Models. • In $[x,y]=\delta x$, $xf(y)=f(y+\delta)x$. If $\delta^2=0$, $[x,f(y)]=\delta f'(y)x$.
• In $[x,y]=\delta x+z^2$, $xf(y)=f(y+\delta)x+\frac{z^2}{\delta}(f(y+\delta)-f(y))$. If $\delta^2=0$, $[x,f(y)]=\delta f'(y)x+z^2f'(y)$.
• If $S_n:=\sum_{k=0}^{n-1}A^kCB^{n-1-k}$ then $AS_n-S_nB=A^nC-CB^n$ so $S_n=(L_A-R_B)^{-1}(A^nC-CB^n)$.
• If $\psi(x)=\sum_{n\geq 0}a_nx^n$ then $\sum_{n\geq 0}a_n\sum_{k=0}^{n-1}b^n(-b)^{n-1-k}=(\psi(b)-\psi(-b))/2b$.

Deriving Gassner. $\mathcal{L}^{2Dw}$ is $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo locality, $[a_{ij},a_{ik}]=0$, $[a_{ik},a_{jk}]=-[a_{ij},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, and $[a_{ij},a_{ji}]=b_ia_{ji}-b_ja_{ij}$. Acts on $V=\mathbb{Q}[\![b_i]\!]\langle x_i=a_{i\infty}\rangle$ by $[a_{ij},x_i]=0$, $[a_{ij},x_j]=b_ix_j-b_jx_i$. Hence $e^{\operatorname{ad}a_{ij}}x_i=x_i$, $e^{\operatorname{ad}a_{ij}}x_j=e^{b_i}x_j+\frac{b_j}{b_i}(1-e^{b_i})x_i$. Renaming $y_i=x_i/b_i$, $t_i=e^{b_i}$ get $[e^{\operatorname{ad}a_{ij}}]_{y_i,y_j}=\begin{pmatrix}1 & 1-t_i\\ 0 & t_i\end{pmatrix}$.

mod $\langle a_{ii}\rangle$

The $\mathcal{L}^{2Dw}$ Adjoint representation. $e^{\operatorname{ad}a_{ij}}$ acts by

$$a_{kl}\mapsto a_{kl},\qquad a_{ik}\mapsto a_{ik},\qquad a_{kj}\mapsto e^{-b_i}a_{kj}+\frac{b_k}{b_i}(1-e^{-b_i})a_{ij},$$

$$a_{ki}\mapsto a_{ki}+(1-e^{-b_i})a_{kj}+b_k\frac{e^{-b_i}-1}{b_i}a_{ij},$$

$$a_{jk}\mapsto e^{b_i}a_{jk}+\frac{b_j}{b_i}(1-e^{b_i})a_{ik},\quad a_{ji}\mapsto e^{b_i}a_{ji}+\frac{b_j}{b_i}(1-e^{b_i})a_{ij}.$$

Adjoint Gassner. Renaming $\alpha_{ij}=a_{ij}/b_i$ and $t_i=e^{b_i}$, get

$$\alpha_{kj}\mapsto t_i^{-1}\alpha_{kj}+(1-t_i^{-1})\alpha_{ij},$$

$$\alpha_{ki}\mapsto a_{ki}+(1-t_i^{-1})\alpha_{kj}+(t_i^{-1}-1)\alpha_{ij}$$

$$\alpha_{jk}\mapsto t_i\alpha_{jk}+(1-t_i)\alpha_{ik},\quad \alpha_{ji}\mapsto t_i\alpha_{ji}+(1-t_i)\alpha_{ij}.$$

Implementation/verification: pensieve://2015-04/nb/ZeroCo.pdf.
Interpretation: π_T-Artin?

2Dv. b: bracket trace; c: cobracket trace; $\langle b,c\rangle=\delta\in\{0,1\}$; $\deg b_i=\deg c_j=\deg a_{ij}=\deg\delta=1$.
$\mathcal{A}^{2Dv}$ is $\mathbb{Q}[\![\delta]\!]FA(b_i,c_j,a_{ij})$ (so $\mathcal{L}^v=\{f+f^{ij}a_{ij}\}$) modulo locality,

Center on jk

tt. $[a_{jk},a_{jl}]=c_la_{jk}-c_ka_{jl}=:\gamma_{jkl}$. (note $\gamma_{j\{kl\}}=0$)
hh. $[a_{jk},a_{ik}]=b_ia_{jk}-b_ja_{ik}$,
th. $[a_{jk},a_{ij}]=b_ja_{ik}-b_ia_{jk}+\gamma_{ijk}$,
⇆. $[a_{ij},a_{ji}]=?$, B
ab, ac. $[a_{ij},b_i]=-[a_{ij},b_j]=-[a_{ij},c_i]=[a_{ij},c_j]=\delta a_{ij}-b_ic_j=:\gamma_{ij}$,
bc. $[b_i,c_j]=0$.
So $a_{ij}f=f^\delta a_{ij}-\frac{b_ic_j}{\delta}(f^\delta-f)$, $[a_{ij},f]=(f^\delta-f)\left(a_{ij}-\frac{b_ic_j}{\delta}\right)$,
with $f^\delta:=f/\!/\begin{pmatrix}b_i\to b_i+\delta & b_j\to b_j-\delta\\ c_i\to c_i-\delta & c_j\to c_j+\delta\end{pmatrix}$.

The Ascending Algebra $\mathcal{A}^{2Dv}_+$. Same but with only a_{ij}, $i<j$.
The OneCo Sub-Quotient is $\langle a_{ij}\rangle$ modulo $\delta^2=\delta c_i=c_jc_k=0$, so $\mathcal{L}^{1co}$ is (coefficient functions non-central, in $\mathbb{Q}[\![b_i]\!]$)

$$\left\{f^{ij}a_{ij}+f^{ijk}\gamma_{ijk}+f^{ijkl}\gamma_{ij}a_{kl}\right\}\Big/(b_i\gamma_{ijk}=\gamma_{ij}a_{ik}-\gamma_{ik}a_{ij})\,.$$

(A)

Then $[a_{ij},f]=(\partial_if-\partial_jf)\gamma_{ij}$ and
γb. $[\gamma_{ij},b_l]=0$ and $[\gamma_{ijk},b_l]=0$ incl. $l\in\{i,j,k\}$,

ttγ. $[a_{jk},\gamma_{jl}]=0$,
hhγ. $[a_{jk},\gamma_{ik}]=-b_j\gamma_{ik}$,
thγ. $[a_{jk},\gamma_{ij}]=b_j\gamma_{ik}$,
htγ. $[a_{jk},\gamma_{kl}]=b_j\gamma_{kl}-b_k\gamma_{jl}$,
ttγ₃. $[a_{jk},\gamma_{jlm}]=0$,
thγ₃. $[a_{jk},\gamma_{ijl}]=b_j\gamma_{ikl}+\gamma_{il}a_{jk}$,
htγ₃. $[a_{jk},\gamma_{klm}]=b_k\gamma_{jkl}+b_j\gamma_{klm}$,
hhγ₃. $[a_{jk},\gamma_{nik}]=-b_j\gamma_{nik}+\gamma_{ni}a_{jk}$,
$[a_{jk},\gamma_{jkl}]=-\gamma_{jk}a_{jl}$,
$[a_{jk},\gamma_{ijk}]=-b_j\gamma_{ijk}+\gamma_{ij}a_{jk}+\gamma_{ik}a_{jk}$.

(Is there a residual 4T?)

~~Specific Brackets~~

A

```
B[a[f_, j_, k_], β[g_]] := γ[f (∂_{b_j} g - ∂_{b_k} g), j, k] // LSimp;

B[a[j_, k_], a[j_, l_]] /; DQ[j, k, l] := γ[1, j, k, l] // LSimp;
B[a[j_, k_], a[i_, k_]] /; DQ[i, j, k] :=
  a[b_i, j, k] - a[b_j, i, k] // LSimp;
B[a[j_, k_], a[i_, j_]] /; DQ[i, j, k] :=
  a[b_j, i, k] - a[b_i, j, k] + γ[1, i, j, k] // LSimp;
B[a[j_, k_], a[k_, l_]] /; DQ[j, k, l] := -B[a[k, l], a[j, k]];
B[a[j_, k_], a[l_, m_]] /; DQ[j, k, l, m] := 0;

B[a[f_, j_, k_], γ[g_, j_, l_]] /; DQ[j, k, l] := 0;
B[a[f_, j_, k_], γ[g_, i_, k_]] /; DQ[i, j, k] :=
  - γ[b_j f g, i, k] // LSimp;
B[a[f_, j_, k_], γ[g_, i_, j_]] /; DQ[i, j, k] := γ[b_j f g, i, k] // LSimp;
B[a[f_, j_, k_], γ[g_, k_, l_]] /; DQ[j, k, l] :=
  γ[b_j f g, k, l] - γ[b_k f g, j, l] // LSimp;
B[a[f_, j_, k_], γ[g_, l_, m_]] /; DQ[j, k, l, m] := 0;
B[a[f_, j_, k_], γ[g_, j_, k_]] := γ[-b_j f g, j, k] // LSimp;

B[a[f_, j_, k_], γ[g_, j_, l_, m_]] /; DQ[j, k, l, m] := 0;
B[a[f_, j_, k_], γ[g_, i_, j_, l_]] /; DQ[i, j, k, l] :=
  γ[b_j f g, i, k, l] + γa[f g, i, l, j, k] // LSimp;
B[a[f_, l_, k_], γ[g_, i_, j_, l_]] /; DQ[i, j, k, l] :=
  γ[-b_l f g, i, k, j] + γa[-f g, i, j, l, k] // LSimp;
B[a[f_, j_, k_], γ[g_, k_, l_, m_]] /; DQ[j, k, l, m] :=
  γ[-b_k f g, j, l, m] + γ[b_j f g, k, l, m] // LSimp;
B[a[f_, j_, k_], γ[g_, n_, i_, k_]] /; DQ[n, i, j, k] :=
  γ[-b_j f g, n, i, k] + γa[f g, n, i, j, k] // LSimp;
B[a[f_, j_, i_], γ[g_, n_, i_, k_]] /; DQ[n, i, j, k] :=
  γ[b_j f g, n, k, i] + γa[-f g, n, k, j, i] // LSimp;
B[a[f_, j_, k_], γ[g_, j_, k_, l_]] /; DQ[j, k, l] :=
  γa[-f g, j, k, j, l] // LSimp;
B[a[f_, j_, l_], γ[g_, j_, k_, l_]] /; DQ[j, k, l] :=
  γa[f g, j, l, j, k] // LSimp;
B[a[f_, j_, k_], γ[g_, i_, j_, k_]] /; DQ[i, j, k] :=
  γ[-b_j f g, i, j, k] + γa[f g, i, j, j, k] + γa[f g, i, k, j, k] // LSimp;
B[a[f_, k_, j_], γ[g_, i_, j_, k_]] /; DQ[i, j, k] :=
  γ[b_k f g, i, k, j] + γa[-f g, i, k, k, j] + γa[-f g, i, j, k, j] // LSimp;
B[a[f_, i_, j_], γ[g_, k_, l_, m_]] /; DQ[i, j, k, l, m] := 0;
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Representations. $\operatorname{ad}a_{-,-}$:

$$\begin{tikzcd}\langle\beta\rangle \arrow[r] & \langle\gamma_{ij}\rangle \arrow[loop right]\end{tikzcd}\qquad \begin{tikzcd}\langle a_{ij}\rangle \arrow[loop left] \arrow[r] \arrow[d] & \langle\gamma_{ijk}\rangle \arrow[loop right] \arrow[dl] \\ \langle\gamma_{ij}a_{kl}\rangle \arrow[loop left] & \end{tikzcd}$$

• $\langle\gamma_{ij}\rangle$ is ....

To do. • Perhaps I should find a way to highlight the fact that v is a perturbation of w. • Position FiC. • Position the 2D Lie bialgebras.

state diagrams for ad a_{jk}: model: what's $\exp\begin{pmatrix} b & \partial \\ 0 & b\end{pmatrix}$?

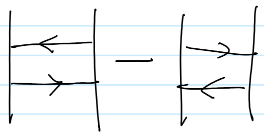
State diagrams for ad a_{jk}:

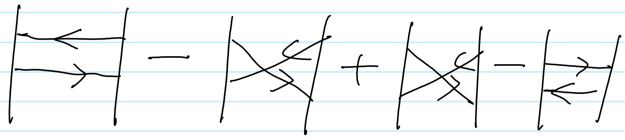
$$a_{jl} \xrightarrow{t} \gamma_{jkl} \xrightarrow{-} \gamma_{jk} a_{jl} \circlearrowleft^{-b_j}$$

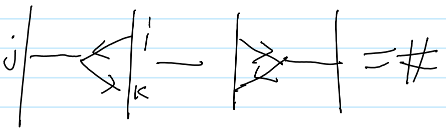
model: What's exp$\begin{pmatrix} & \\ 0 & b\end{pmatrix}$?

$$\begin{tikzcd} fa_{ik} \arrow[loop above, "-b_j"] \arrow[r, "b_i"] \arrow[d, "\partial_j-\partial_k"] & fa_{jk} \arrow[d, "\partial_j-\partial_k"] \\ f\gamma_{jk}a_{ik} \arrow[r, "b_i"] \arrow[loop below, "-2b_j"] & f\gamma_{jk}a_{jk} \end{tikzcd}$$

B.

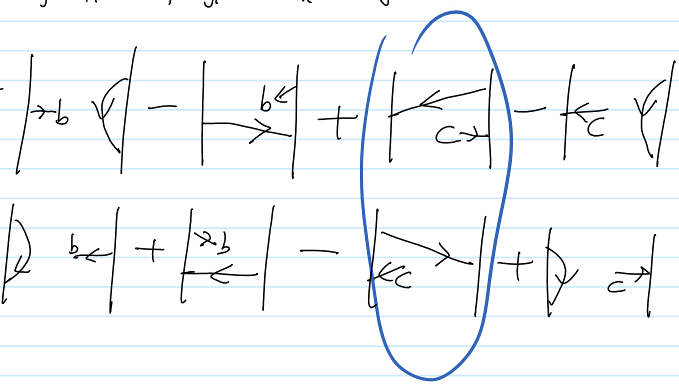
$[a_{jk}, a_{kj}] =$ 

$=$ 

$=$  $= \#$

$\downarrow$

$$[a_{jk}, a_{ij}] = b_j a_{ik} - b_i a_{jk} + \gamma_{ijk}$$
$$= b_j a_{ik} - b_i a_{jk} + c_k a_{ij} - c_j a_{ik}$$

$\# =$ 

$=$

might be that I need to allow γ_{ijk}