Pensieve header: γ3-checks for OneCo. Branched from OneCoComputations.nb.

The bracket

In the basis {a[f,i,j], γ[f,i,j], γ[f,i,j,k], γa[f,i,j,k,l]}.

```
DistinctQ[is___] := (Sort[{is}] === Union[{is}]);

a[0, __] := 0;
a /: a[f_, is__] + a[g_, is__] := a[f + g, is];
a /: -a[f_, is__] := a[Expand[-f], is];
γ[0, __] := 0;
γ /: γ[f_, is__] + γ[g_, is__] := γ[f + g, is];
γ /: -γ[f_, is__] := γ[Expand[-f], is];
γa[0, __] := 0;
γa /: γa[f_, is__] + γa[g_, is__] := γa[f + g, is];
γa /: -γa[f_, is__] := γa[Expand[-f], is];

γ[f_, j_, k_, k_] = 0;
γ[f_, j_, k_, l_] /; OrderedQ[{l, k}] ⋀ DistinctQ[k, l] := γ[-f, j, l, k];

B[0, _] = 0; B[_, 0] = 0;
B[x_, x_] = 0;
B[x_Plus, y_] := B[#, y] & /@ x;
B[x_, y_Plus] := B[x, #] & /@ y;
```

[a,a] brackets:

```
B[a[f_, j_, k_], a[g_, l_, m_]] := Plus[
   B[a[j, k], a[l, m]] /.
    {a[h_, is__] :> a[Expand[f g h], is], γ[h_, is__] :> γ[Expand[f g h], is]},
   γa[Expand[f (∂b_j g - ∂b_k g)], j, k, l, m] + γa[Expand[g (∂b_m f - ∂b_l f)], j, k, l, m]
  ];
(* tt *) B[a[j_, k_], a[j_, l_]] /; DistinctQ[j, k, l] := γ[1, j, k, l];
(* hh *) B[a[j_, k_], a[i_, k_]] /; DistinctQ[i, j, k] := a[b_i, j, k] - a[b_j, i, k];
(* th *) B[a[j_, k_], a[i_, j_]] /; DistinctQ[i, j, k] :=
  a[b_j, i, k] - a[b_i, j, k] + γ[1, i, j, k];
(* ht *) B[a[j_, k_], a[k_, l_]] /; DistinctQ[j, k, l] := -B[a[k, l], a[j, k]];
(* loc *) B[a[j_, k_], a[l_, m_]] /; DistinctQ[j, k, l, m] := 0;
```

[a, γ_2] brackets:

```
B[γ[f_, i_, j_], a[g_, k_, l_]] := -B[a[g, k, l], γ[f, i, j]];
(* ttγ *) B[a[f_, j_, k_], γ[g_, j_, l_]] /; DistinctQ[j, k, l] := 0;
(* hhγ *) B[a[f_, j_, k_], γ[g_, i_, k_]] /; DistinctQ[i, j, k] := -γ[b_j f g, i, k];
(* thγ *) B[a[f_, j_, k_], γ[g_, i_, j_]] /; DistinctQ[i, j, k] := γ[b_j f g, i, k];
(* htγ *) B[a[f_, j_, k_], γ[g_, k_, l_]] /; DistinctQ[j, k, l] :=
  γ[b_j f g, k, l] - γ[b_j f g, j, l];
(* locγ *) B[a[f_, j_, k_], γ[g_, l_, m_]] /; DistinctQ[j, k, l, m] := 0;
B[a[f_, j_, k_], γ[g_, j_, k_]] := γ[-b_j f g, j, k];
```

[a, γ_3] brackets:

```
(* ttγ3 *) B[a[f_, j_, k_], γ[g_, j_, l_, m_]] /; DistinctQ[j, k, l, m] := 0;
(* thγ3-1 *) B[a[f_, j_, k_], γ[g_, i_, j_, l_]] /; DistinctQ[i, j, k, l] :=
  γ[b_j f g, i, k, l] + γa[f g, i, l, j, k];
(* thγ3-2 *) B[a[f_, l_, k_], γ[g_, i_, j_, l_]] /; DistinctQ[i, j, k, l] :=
  γ[-b_l f g, i, k, j] + γa[-f g, i, j, l, k];
(* htγ3 *) B[a[f_, j_, k_], γ[g_, k_, l_, m_]] /; DistinctQ[j, k, l, m] :=
  γ[-b_k f g, j, l, m] + γ[b_j f g, k, l, m];
(* hhγ3-1 *) B[a[f_, j_, k_], γ[g_, n_, i_, k_]] /; DistinctQ[n, i, j, k] :=
  γ[-b_j f g, n, i, k] + γa[f g, n, i, j, k];
(* hhγ3-2 *) B[a[f_, j_, i_], γ[g_, n_, i_, k_]] /; DistinctQ[n, i, j, k] :=
  γ[b_j f g, n, k, i] + γa[-f g, n, k, j, i];
(* a-th-1 *) B[a[f_, j_, k_], γ[g_, j_, k_, l_]] /; DistinctQ[j, k, l] :=
  γa[-f g, j, k, j, l];
(* a-th-2 *) B[a[f_, j_, l_], γ[g_, j_, k_, l_]] /; DistinctQ[j, k, l] :=
  γa[f g, j, l, j, k];
(* a-hh-1 *) B[a[f_, j_, k_], γ[g_, i_, j_, k_]] /; DistinctQ[i, j, k] :=
  γ[-b_j f g, i, j, k] + γa[f g, i, j, j, k] + γa[f g, i, k, j, k];
(* a-hh-2 *) B[a[f_, k_, j_], γ[g_, i_, j_, k_]] /; DistinctQ[i, j, k] :=
  γ[b_k f g, i, k, j] + γa[-f g, i, k, k, j] + γa[-f g, i, j, k, j];
```

[γ, γ] brackets:

```
B[_γ, _γ] := 0;
```

γ_3 checks

```
ca[0, __] := 0;
ca /: ca[f_, is__] + ca[g_, is__] := ca[f + g, is];
ca /: -ca[f_, is__] := ca[Expand[-f], is];
B[a[f_, j_, k_], c[l_]] /; DistinctQ[j, k, l] := 0;
B[a[f_, j_, k_], c[j_]] := γ[-f, j, k];
B[a[f_, j_, k_], c[k_]] := γ[f, j, k];
B[x_, ca[f_, i_, j_, k_]] := Plus[
   B[x, c[i]] /. γ[g_, l_, m_] :> γa[f g, l, m, j, k],
   B[x, a[f, j, k]] /. {a[g_, l_, m_] :> ca[g, i, l, m], _γ → 0, _γa → 0}
  ];
δaa[f_, i_, j_, k_, l_] /; OrderedQ[{l, j}] ⋀ DistinctQ[i, j, k, l] :=
  δaa[f, k, l, i, j];
δaa[f_, i_, j_, i_, l_] /; OrderedQ[{l, j}] ⋀ DistinctQ[j, l] := δaa[f, i, l, i, j];
(* missing a 4T! *)
Eγ = {
   γ[f_, j_, k_, l_] :> ca[f, l, j, k] + ca[Expand[-f], k, j, l],
   γa[f_, i_, j_, k_, l_] :> δaa[f, i, j, k, l] + ca[Expand[-bi f], j, k, l]
  };
Check3[y_γ] :=
 {t1 = B[a[f, j, k], y] /. Eγ, t2 = B[a[f, j, k], y /. Eγ] /. Eγ, t1 == t2 // Simplify}
```

tt check

```
γ[g, j, l, m] // Check3
```

```
{0, 0, True}
```

th checks

```
γ[g, i, j, l] // Check3 // ColumnForm
```

```
ca[-f g bi, l, j, k] + ca[-f g bj, k, i, l] + ca[f g bj, l, i, k] + δaa[f g, j, k, i, l]
ca[-f g bi, l, j, k] + ca[-f g bj, k, i, l] + ca[f g bj, l, i, k] + δaa[f g, j, k, i, l]
True
```

```
γ[g, i, l, j] // Check3 // ColumnForm
```

```
ca[f g bi, l, j, k] + ca[-f g bj, l, i, k] + ca[f g bj, k, i, l] + δaa[-f g, j, k, i, l]
ca[f g bi, l, j, k] + ca[-f g bj, l, i, k] + ca[f g bj, k, i, l] + δaa[-f g, j, k, i, l]
True
```

ht check

```
γ[g, k, l, m] // Check3 // ColumnForm
```

```
ca[-f g bj, l, k, m] + ca[f g bj, m, k, l] + ca[-f g bk, m, j, l] + ca[f g bk, l, j, m]
ca[-f g bj, l, k, m] + ca[f g bj, m, k, l] + ca[-f g bk, m, j, l] + ca[f g bk, l, j, m]
True
```

hh checks

```
γ[g, n, i, k] // Check3 // ColumnForm
```

```
ca[-f g b_j, k, n, i] + ca[f g b_j, i, n, k] + ca[-f g b_n, i, j, k] + δaa[f g, n, i, j, k]
ca[-f g b_j, k, n, i] + ca[f g b_j, i, n, k] + ca[-f g b_n, i, j, k] + δaa[f g, n, i, j, k]
True
```

```
γ[g, n, k, i] // Check3 // ColumnForm
```

```
ca[-f g b_j, i, n, k] + ca[f g b_j, k, n, i] + ca[f g b_n, i, j, k] + δaa[-f g, n, i, j, k]
ca[-f g b_j, i, n, k] + ca[f g b_j, k, n, i] + ca[f g b_n, i, j, k] + δaa[-f g, n, i, j, k]
True
```

a-th checks

```
γ[g, j, l, k] // Check3 // ColumnForm
```

```
ca[-f g b_j, k, j, l] + δaa[f g, j, k, j, l]
ca[-f g b_j, k, j, l] + δaa[f g, j, k, j, l]
True
```

```
γ[g, j, l, k] // Check3 // ColumnForm
```

```
ca[-f g b_j, k, j, l] + δaa[f g, j, k, j, l]
ca[-f g b_j, k, j, l] + δaa[f g, j, k, j, l]
True
```

a-hh checks

```
γ[g, i, j, k] // Check3 // ColumnForm
```

```
ca[-f g b_i, j, j, k] + ca[-f g b_i, k, j, k] + ca[-f g b_j, k, i, j] + ca[f g b_j, j, i, k] + δaa[f g,
ca[-f g b_i, j, j, k] + ca[-f g b_i, k, j, k] + ca[-f g b_j, k, i, j] + ca[f g b_j, j, i, k] + δaa[f g,
δaa[f g, i, j, j, k] + δaa[f g, i, k, j, k] == δaa[f g, j, k, i, j] + δaa[f g, j, k, i, k]
```

```
γ[g, i, k, j] // Check3 // ColumnForm
```

```
ca[f g b_i, j, j, k] + ca[f g b_i, k, j, k] + ca[-f g b_j, j, i, k] + ca[f g b_j, k, i, j] + δaa[-f g, i
ca[f g b_i, j, j, k] + ca[f g b_i, k, j, k] + ca[-f g b_j, j, i, k] + ca[f g b_j, k, i, j] + δaa[-f g, j
δaa[-f g, i, j, j, k] + δaa[-f g, i, k, j, k] == δaa[-f g, j, k, i, j] + δaa[-f g, j, k, i, k]
```

---

x1 - x2 - x3 check

```
{x1, x2, x3} = {a[f[b1, b2, b3], 1, 2],
   a[g[b1, b2, b3], 1, 3], a[h[b1, b2, b3], 2, 3]} /. {_f | _g | _h :> 1}
```

```
{a[1, 1, 2], a[1, 1, 3], a[1, 2, 3]}
```

```
B[x2, x3] /. Eγ
```

```
a[-b1, 2, 3] + a[b2, 1, 3]
```

```
j1 = B[x1, B[x2, x3] /. Eγ] /. Eγ /. _δaa :> 0
```

```
a[-b1^2, 2, 3] + a[b1 b2, 1, 3] + ca[b1, 2, 2, 3] + ca[-b2, 2, 1, 3] + ca[b1 + b2, 3, 1, 2]
```

```
j2 = B[x2, B[x3, x1] /. Eγ] /. Eγ /. _δaa → 0
```

```
a[b1^2, 2, 3] + a[-b1 b2, 1, 3] + ca[-b1, 3, 1, 2] + ca[b1, 3, 2, 3]
```

```
j3 = B[x3, B[x1, x2] /. Eγ] /. Eγ /. _δaa → 0
```

$$ca[-b_1, 2, 2, 3] + ca[-b_1, 3, 2, 3] + ca[-b_2, 3, 1, 2] + ca[b_2, 2, 1, 3]$$

```
j1 + j2 + j3
```

$$0$$

```
B[x1, B[x2, x3]] /. Eγ /. _δaa :→ 0
```

$$a\left[-b_1^2, 2, 3\right] + a[b_1\, b_2, 1, 3] + ca[b_1, 2, 2, 3] + ca[-b_2, 2, 1, 3] + ca[b_1 + b_2, 3, 1, 2]$$

```
B[x2, B[x3, x1]] /. Eγ /. _δaa → 0
```

$$a\left[b_1^2, 2, 3\right] + a[-b_1\, b_2, 1, 3] + ca[-b_1, 3, 1, 2] + ca[b_1, 3, 2, 3]$$

```
B[x3, B[x1, x2]] /. Eγ /. _δaa → 0
```

$$ca[-b_1, 2, 2, 3] + ca[-b_1, 3, 2, 3] + ca[-b_2, 3, 1, 2] + ca[b_2, 2, 1, 3]$$

```
(B[x1, B[x2, x3]] + B[x2, B[x3, x1]] + B[x3, B[x1, x2]])
```

$$\gamma[b_1, 1, 2, 3] + \gamma a[-1, 1, 2, 1, 3] + \gamma a[1, 1, 3, 1, 2]$$

```
(B[x1, B[x2, x3]] + B[x2, B[x3, x1]] + B[x3, B[x1, x2]]) /. Eγ
```

$$\delta aa[-1, 1, 2, 1, 3] + \delta aa[1, 1, 2, 1, 3]$$

```
γ[b1, 1, 2, 3] /. Eγ
```

$$ca[-b_1, 2, 1, 3] + ca[b_1, 3, 1, 2]$$