

$$\begin{aligned}
 s1 &= a\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, 2, 4\right] // \text{Ad}[a[s, 2, 3]] // \text{LSimp} \\
 &\left\{\beta\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}\right] + \gamma\left[\frac{(-1 + e^{-s b_2})\left(\frac{1}{b_2} - \frac{e^{t b_2}}{b_2} + \frac{b_3}{b_2^2} - \frac{e^{t b_2} b_3}{b_2^2} + \frac{e^{t b_2} t b_3}{b_2}\right)}{b_2}, 2, 3\right], \frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, \text{Null}\right\} \\
 &\left\{\gamma\left[\frac{(-1 + e^{-s b_2})\left(\frac{1}{b_2} - \frac{e^{t b_2}}{b_2} + \frac{b_3}{b_2^2} - \frac{e^{t b_2} b_3}{b_2^2} + \frac{e^{t b_2} t b_3}{b_2}\right)}{b_2}, 2, 3\right]\right\} \\
 &a\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, 2, 4\right] + \gamma\left[\frac{s b_3}{b_2} - \frac{e^{t b_2} s b_3}{b_2}, 2, 3, 4\right] + \\
 &\gamma a\left[-\frac{1}{b_2^2} + \frac{e^{-s b_2}}{b_2^2} + \frac{e^{t b_2}}{b_2^2} - \frac{e^{-s b_2+t b_2}}{b_2^2} - \frac{s b_3}{b_2^2} + \frac{e^{t b_2} s b_3}{b_2^2} - \frac{e^{t b_2} t b_3}{b_2^2} + \frac{e^{-s b_2+t b_2} t b_3}{b_2^2}, 2, 3, 2, 4\right]
 \end{aligned}$$

$$\begin{aligned}
 s2 &= a\left[\frac{(1 - e^{t b_2}) b_3}{b_2}, 2, 4\right] // \text{Ad}[a[s, 2, 3]] // \text{LSimp} \\
 &\left\{\beta\left[\frac{(1 - e^{t b_2}) b_3}{b_2}\right] + \gamma\left[\frac{(-1 + e^{-s b_2})\left(\frac{1 - e^{t b_2}}{b_2} + \frac{(1 - e^{t b_2}) b_3}{b_2^2} + \frac{e^{t b_2} t b_3}{b_2}\right)}{b_2}, 2, 3\right], \frac{(1 - e^{t b_2}) b_3}{b_2}, \text{Null}\right\} \\
 &\left\{\gamma\left[\frac{(-1 + e^{-s b_2})\left(\frac{1 - e^{t b_2}}{b_2} + \frac{(1 - e^{t b_2}) b_3}{b_2^2} + \frac{e^{t b_2} t b_3}{b_2}\right)}{b_2}, 2, 3\right]\right\} \\
 &a\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, 2, 4\right] + \gamma\left[\frac{s b_3}{b_2} - \frac{e^{t b_2} s b_3}{b_2}, 2, 3, 4\right] + \\
 &\gamma a\left[-\frac{1}{b_2^2} + \frac{e^{-s b_2}}{b_2^2} + \frac{e^{t b_2}}{b_2^2} - \frac{e^{-s b_2+t b_2}}{b_2^2} - \frac{s b_3}{b_2^2} + \frac{e^{t b_2} s b_3}{b_2^2} - \frac{e^{t b_2} t b_3}{b_2^2} + \frac{e^{-s b_2+t b_2} t b_3}{b_2^2}, 2, 3, 2, 4\right]
 \end{aligned}$$

s1 - s2 // LSimp

0

$$\begin{aligned}
 &\text{Simplify}\left[\frac{(-1 + e^{-s b_2})\left(\frac{1 - e^{t b_2}}{b_2} + \frac{(1 - e^{t b_2}) b_3}{b_2^2} + \frac{e^{t b_2} t b_3}{b_2}\right)}{b_2}\right] \\
 &= \frac{e^{-s b_2} (-1 + e^{s b_2}) \left(-(-1 + e^{t b_2}) b_3 + b_2 (1 - e^{t b_2} + e^{t b_2} t b_3)\right)}{b_2^3}
 \end{aligned}$$

t0 = a[g, 3, 4]

a[g, 3, 4]

{t, s}

{t, s}

t0 // Ad[a[t, 2, 3]]

$$a\left[e^{t b_2} g, 3, 4\right] + a\left[\frac{g b_3}{b_2} - \frac{e^{t b_2} g b_3}{b_2}, 2, 4\right] +$$

$$\gamma\left[\frac{g}{b_2} - \frac{e^{t b_2} g}{b_2} + \frac{g b_3}{b_2^2} - \frac{e^{t b_2} g b_3}{b_2^2} + \frac{g t b_3}{b_2}, 2, 3, 4\right] + \gamma a\left[\frac{g}{b_2^2} - \frac{e^{t b_2} g}{b_2^2} + \frac{e^{t b_2} g t}{b_2}, 2, 3, 3, 4\right] +$$

$$\gamma a\left[-\frac{2 g}{b_2^2} + \frac{e^{-t b_2} g}{b_2^2} + \frac{e^{t b_2} g}{b_2^2} - \frac{2 g b_3}{b_2^3} + \frac{2 e^{t b_2} g b_3}{b_2^3} - \frac{g t b_3}{b_2^2} - \frac{e^{t b_2} g t b_3}{b_2^2}, 2, 3, 2, 4\right]$$

((t0 // Ad[a[t, 2, 3]]) /. g -> 1) - ((t0 /. g -> 1) // Ad[a[t, 2, 3]])

0

(tt = {((t0 // Ad[a[t, 2, 3]]) /. g -> 1), ((t0 /. g -> 1) // Ad[a[t, 2, 3]])}) //

ColumnForm

$$a\left[e^{t b_2}, 3, 4\right] + a\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, 2, 4\right] + \gamma\left[\frac{1}{b_2} - \frac{e^{t b_2}}{b_2} + \frac{b_3}{b_2^2} - \frac{e^{t b_2} b_3}{b_2^2} + \frac{t b_3}{b_2}, 2, 3, 4\right] + \gamma a\left[\frac{1}{b_2^2} - \frac{e^{t b_2}}{b_2^2} + \frac{e^{t b_2} t}{b_2}, 2, 3, 3, 4\right]$$

$$a\left[e^{t b_2}, 3, 4\right] + a\left[\frac{(1-e^{t b_2}) b_3}{b_2}, 2, 4\right] + \gamma\left[\frac{t b_2 b_3 + (1-e^{t b_2})(b_2+b_3)}{b_2^2}, 2, 3, 4\right] + \gamma a\left[\frac{1+e^{t b_2}(-1+t b_2)}{b_2^2}, 2, 3, 3, 4\right]$$

{1, -1}.tt

0

tt[[All, 2]]

$$\left\{a\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, 2, 4\right], a\left[\frac{(1-e^{t b_2}) b_3}{b_2}, 2, 4\right]\right\}$$

(ss = (#[[2]] // Ad[a[s, 2, 3]] // LSimp) & /@ tt) // ColumnForm

$$a\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, 2, 4\right] + \gamma\left[\frac{s b_3}{b_2} - \frac{e^{t b_2} s b_3}{b_2}, 2, 3, 4\right] + \gamma a\left[-\frac{1}{b_2^2} + \frac{e^{-s b_2}}{b_2^2} + \frac{e^{t b_2}}{b_2^2} - \frac{e^{-s b_2+t b_2}}{b_2^2} - \frac{s b_3}{b_2^2} + \frac{e^{t b_2} s b_3}{b_2^2} - \frac{e^{t b_2} t b_3}{b_2^2}\right]$$

$$a\left[\frac{b_3}{b_2} - \frac{e^{t b_2} b_3}{b_2}, 2, 4\right] + \gamma\left[\frac{s b_3}{b_2} - \frac{e^{t b_2} s b_3}{b_2}, 2, 3, 4\right] + \gamma a\left[\frac{b_3}{b_2^2} - \frac{e^{-s b_2} b_3}{b_2^2} - \frac{e^{t b_2} b_3}{b_2^2} + \frac{e^{-s b_2+t b_2} b_3}{b_2^2} - \frac{s b_3}{b_2^2} + \frac{e^{t b_2} s b_3}{b_2^2}, 2, 3, 3, 4\right]$$

{1, -1}.ss

$$\gamma a\left[-\frac{1}{b_2^2} + \frac{e^{-s b_2}}{b_2^2} + \frac{e^{t b_2}}{b_2^2} - \frac{e^{-s b_2+t b_2}}{b_2^2} - \frac{b_3}{b_2^3} + \frac{e^{-s b_2} b_3}{b_2^3} + \frac{e^{t b_2} b_3}{b_2^3} - \frac{e^{-s b_2+t b_2} b_3}{b_2^3} - \frac{e^{t b_2} t b_3}{b_2^2} + \frac{e^{-s b_2+t b_2} t b_3}{b_2^2}, 2, 3, 2, 4\right]$$

t1 = (t0 /. g -> 1) // Ad[a[t, 2, 3]] // Ad[a[s, 2, 3]] // LSimp

$$\begin{aligned}
 & a[e^{s b_2+t b_2}, 3, 4] + a\left[\frac{b_3}{b_2} - \frac{e^{s b_2+t b_2} b_3}{b_2}, 2, 4\right] + \\
 & \gamma\left[\frac{1}{b_2} - \frac{e^{s b_2+t b_2}}{b_2} + \frac{b_3}{b_2^2} - \frac{e^{s b_2+t b_2} b_3}{b_2^2} + \frac{s b_3}{b_2} + \frac{t b_3}{b_2}, 2, 3, 4\right] + \\
 & \gamma a\left[\frac{1}{b_2^2} - \frac{e^{s b_2+t b_2}}{b_2^2} + \frac{e^{s b_2+t b_2} s}{b_2} + \frac{e^{s b_2+t b_2} t}{b_2}, 2, 3, 3, 4\right] + \\
 & \gamma a\left[-\frac{1}{b_2^2} - \frac{e^{-s b_2}}{b_2^2} - \frac{e^t b_2}{b_2^2} + \frac{e^{-s b_2-t b_2}}{b_2^2} + \frac{e^{-s b_2+t b_2}}{b_2^2} + \frac{e^{s b_2+t b_2}}{b_2^2} - \frac{b_3}{b_2^3} - \right. \\
 & \quad \left. \frac{e^{-s b_2} b_3}{b_2^3} - \frac{e^t b_2 b_3}{b_2^3} + \frac{e^{-s b_2+t b_2} b_3}{b_2^3} + \frac{2 e^{s b_2+t b_2} b_3}{b_2^3} - \frac{s b_3}{b_2^2} - \frac{e^{s b_2+t b_2} s b_3}{b_2^2} - \right. \\
 & \quad \left. \frac{t b_3}{b_2^2} + \frac{e^t b_2 t b_3}{b_2^2} - \frac{e^{-s b_2+t b_2} t b_3}{b_2^2} - \frac{e^{s b_2+t b_2} t b_3}{b_2^2}, 2, 3, 2, 4\right]
 \end{aligned}$$

t2 = ((t0 // Ad[a[t, 2, 3]]) /. g -> 1) // Ad[a[s, 2, 3]] // LSimp

$$\begin{aligned}
 & a[e^{s b_2+t b_2}, 3, 4] + a\left[\frac{b_3}{b_2} - \frac{e^{s b_2+t b_2} b_3}{b_2}, 2, 4\right] + \\
 & \gamma\left[\frac{1}{b_2} - \frac{e^{s b_2+t b_2}}{b_2} + \frac{b_3}{b_2^2} - \frac{e^{s b_2+t b_2} b_3}{b_2^2} + \frac{s b_3}{b_2} + \frac{t b_3}{b_2}, 2, 3, 4\right] + \\
 & \gamma a\left[\frac{1}{b_2^2} - \frac{e^{s b_2+t b_2}}{b_2^2} + \frac{e^{s b_2+t b_2} s}{b_2} + \frac{e^{s b_2+t b_2} t}{b_2}, 2, 3, 3, 4\right] + \gamma a\left[-\frac{2}{b_2^2} + \frac{e^{-s b_2-t b_2}}{b_2^2} + \frac{e^{s b_2+t b_2}}{b_2^2} - \right. \\
 & \quad \left. \frac{2 b_3}{b_2^3} + \frac{2 e^{s b_2+t b_2} b_3}{b_2^3} - \frac{s b_3}{b_2^2} - \frac{e^{s b_2+t b_2} s b_3}{b_2^2} - \frac{t b_3}{b_2^2} - \frac{e^{s b_2+t b_2} t b_3}{b_2^2}, 2, 3, 2, 4\right]
 \end{aligned}$$

LSimp[t1 - t2]

$$\begin{aligned}
 & \gamma a\left[\frac{1}{b_2^2} - \frac{e^{-s b_2}}{b_2^2} - \frac{e^t b_2}{b_2^2} + \frac{e^{-s b_2+t b_2}}{b_2^2} + \frac{b_3}{b_2^3} - \frac{e^{-s b_2} b_3}{b_2^3} - \right. \\
 & \quad \left. \frac{e^t b_2 b_3}{b_2^3} + \frac{e^{-s b_2+t b_2} b_3}{b_2^3} + \frac{e^t b_2 t b_3}{b_2^3} - \frac{e^{-s b_2+t b_2} t b_3}{b_2^3}, 2, 3, 2, 4\right]
 \end{aligned}$$

Expand[(((# // Ad[a[s, 2, 3]]) /. g -> 1) == ((# /. g -> 1) // Ad[a[s, 2, 3]])] & /@ (List @@ (a[g, 3, 4] // Ad[a[t, 2, 3]])]

{True, True, True, True, True}