

```
{x, y} = {a[t, j, k], a[1, k, 1]}
```

```
{a[t, j, k], a[1, k, 1]}
```

```
pows = NestList[B[x, #] &, y, 15]
```

```
{a[1, k, 1], a[t bj, k, 1] + a[-t bk, j, 1] + γ[-t, j, k, 1],
 a[t2 bj2, k, 1] + a[-t2 bj bk, j, 1] + γ[-t2 bj - t2 bk, j, k, 1] + γa[t2, j, k, k, 1] +
  γa[2 t2, j, k, j, 1], a[t3 bj3, k, 1] + a[-t3 bj2 bk, j, 1] + γ[-t3 bj2 - t3 bj bk, j, k, 1] +
  γa[2 t3 bj, j, k, k, 1] + γa[-2 t3 bj + t3 (bj - bk) - t3 bk + t3 (bj + bk), j, k, j, 1],
 a[t4 bj4, k, 1] + a[-t4 bj3 bk, j, 1] + γ[-t4 bj3 - t4 bj2 bk, j, k, 1] +
  γa[3 t4 bj2, j, k, k, 1] + γa[t4 bj (bj - 2 bk) - t4 bj bk + t4 bj (bj + bk), j, k, j, 1],
 a[t5 bj5, k, 1] + a[-t5 bj4 bk, j, 1] + γ[-t5 bj4 - t5 bj3 bk, j, k, 1] + γa[4 t5 bj3, j, k, k, 1] +
  γa[t5 bj2 (bj - 3 bk) - 3 t5 bj2 bk + 2 t5 bj2 (-bj + bk) + t5 bj2 (bj + bk), j, k, j, 1],
 a[t6 bj6, k, 1] + a[-t6 bj5 bk, j, 1] + γ[-t6 bj5 - t6 bj4 bk, j, k, 1] +
  γa[5 t6 bj4, j, k, k, 1] + γa[t6 bj3 (bj - 4 bk) - t6 bj3 bk + t6 bj3 (bj + bk), j, k, j, 1],
 a[t7 bj7, k, 1] + a[-t7 bj6 bk, j, 1] + γ[-t7 bj6 - t7 bj5 bk, j, k, 1] + γa[6 t7 bj5, j, k, k, 1] +
  γa[t7 bj4 (bj - 5 bk) - 2 t7 bj4 (bj - 2 bk) - 5 t7 bj4 bk + t7 bj4 (bj + bk), j, k, j, 1],
 a[t8 bj8, k, 1] + a[-t8 bj7 bk, j, 1] + γ[-t8 bj7 - t8 bj6 bk, j, k, 1] +
  γa[7 t8 bj6, j, k, k, 1] + γa[t8 bj5 (bj - 6 bk) - t8 bj5 bk + t8 bj5 (bj + bk), j, k, j, 1],
 a[t9 bj9, k, 1] + a[-t9 bj8 bk, j, 1] + γ[-t9 bj8 - t9 bj7 bk, j, k, 1] + γa[8 t9 bj7, j, k, k, 1] +
  γa[t9 bj6 (bj - 7 bk) - 2 t9 bj6 (bj - 3 bk) - 7 t9 bj6 bk + t9 bj6 (bj + bk), j, k, j, 1],
 a[t10 bj10, k, 1] + a[-t10 bj9 bk, j, 1] + γ[-t10 bj9 - t10 bj8 bk, j, k, 1] +
  γa[9 t10 bj8, j, k, k, 1] + γa[t10 bj7 (bj - 8 bk) - t10 bj7 bk + t10 bj7 (bj + bk), j, k, j, 1],
 a[t11 bj11, k, 1] + a[-t11 bj10 bk, j, 1] + γ[-t11 bj10 - t11 bj9 bk, j, k, 1] + γa[10 t11 bj9, j, k,
  k, 1] + γa[t11 bj8 (bj - 9 bk) - 2 t11 bj8 (bj - 4 bk) - 9 t11 bj8 bk + t11 bj8 (bj + bk), j, k, j, 1],
 a[t12 bj12, k, 1] + a[-t12 bj11 bk, j, 1] + γ[-t12 bj11 - t12 bj10 bk, j, k, 1] +
  γa[11 t12 bj10, j, k, k, 1] + γa[t12 bj9 (bj - 10 bk) - t12 bj9 bk + t12 bj9 (bj + bk), j, k, j, 1],
 a[t13 bj13, k, 1] + a[-t13 bj12 bk, j, 1] + γ[-t13 bj12 - t13 bj11 bk, j, k, 1] +
  γa[12 t13 bj11, j, k, k, 1] +
  γa[t13 bj10 (bj - 11 bk) - 2 t13 bj10 (bj - 5 bk) - 11 t13 bj10 bk + t13 bj10 (bj + bk), j, k, j, 1],
 a[t14 bj14, k, 1] + a[-t14 bj13 bk, j, 1] + γ[-t14 bj13 - t14 bj12 bk, j, k, 1] +
  γa[13 t14 bj12, j, k, k, 1] + γa[t14 bj11 (bj - 12 bk) - t14 bj11 bk + t14 bj11 (bj + bk), j, k, j, 1],
 a[t15 bj15, k, 1] + a[-t15 bj14 bk, j, 1] + γ[-t15 bj14 - t15 bj13 bk, j, k, 1] +
  γa[14 t15 bj13, j, k, k, 1] +
  γa[t15 bj12 (bj - 13 bk) - 2 t15 bj12 (bj - 6 bk) - 13 t15 bj12 bk + t15 bj12 (bj + bk), j, k, j, 1]}
```

```
states = Union[Cases[pows, s_a | s_β | s_γ | s_γα => ReplacePart[s, 1 -> _], ∞]]
```

```
{a[_ , j, 1], a[_ , k, 1], γ[_ , j, k, 1], γa[_ , j, k, j, 1], γa[_ , j, k, k, 1]}
```

```
s = γ[_ , j, k, 1]
```

```
γ[_ , j, k, 1]
```

```
seq = Cases[{#}, s, ∞] & /@ pows
```

```
{ {}, {γ[-t, j, k, 1]}, {γ[-t2 bj - t2 bk, j, k, 1]}, {γ[-t3 bj2 - t3 bj bk, j, k, 1]},  
{γ[-t4 bj3 - t4 bj2 bk, j, k, 1]}, {γ[-t5 bj4 - t5 bj3 bk, j, k, 1]}, {γ[-t6 bj5 - t6 bj4 bk, j, k, 1]},  
{γ[-t7 bj6 - t7 bj5 bk, j, k, 1]}, {γ[-t8 bj7 - t8 bj6 bk, j, k, 1]}, {γ[-t9 bj8 - t9 bj7 bk, j, k, 1]},  
{γ[-t10 bj9 - t10 bj8 bk, j, k, 1]}, {γ[-t11 bj10 - t11 bj9 bk, j, k, 1]},  
{γ[-t12 bj11 - t12 bj10 bk, j, k, 1]}, {γ[-t13 bj12 - t13 bj11 bk, j, k, 1]},  
{γ[-t14 bj13 - t14 bj12 bk, j, k, 1]}, {γ[-t15 bj14 - t15 bj13 bk, j, k, 1]} }
```

```
sh = 0; While[seq[[1]] == {}, ++sh; seq = Rest[seq]]; {sh, seq}
```

```
{1, { {γ[-t, j, k, 1]}, {γ[-t2 bj - t2 bk, j, k, 1]}, {γ[-t3 bj2 - t3 bj bk, j, k, 1]},  
{γ[-t4 bj3 - t4 bj2 bk, j, k, 1]}, {γ[-t5 bj4 - t5 bj3 bk, j, k, 1]},  
{γ[-t6 bj5 - t6 bj4 bk, j, k, 1]}, {γ[-t7 bj6 - t7 bj5 bk, j, k, 1]},  
{γ[-t8 bj7 - t8 bj6 bk, j, k, 1]}, {γ[-t9 bj8 - t9 bj7 bk, j, k, 1]},  
{γ[-t10 bj9 - t10 bj8 bk, j, k, 1]}, {γ[-t11 bj10 - t11 bj9 bk, j, k, 1]},  
{γ[-t12 bj11 - t12 bj10 bk, j, k, 1]}, {γ[-t13 bj12 - t13 bj11 bk, j, k, 1]},  
{γ[-t14 bj13 - t14 bj12 bk, j, k, 1]}, {γ[-t15 bj14 - t15 bj13 bk, j, k, 1]} } }
```

```
seq = Replace[seq, {{_ [f_, ___]} => f, {} -> 0}, {1}]; seq
```

```
{-t, -t2 bj - t2 bk, -t3 bj2 - t3 bj bk, -t4 bj3 - t4 bj2 bk, -t5 bj4 - t5 bj3 bk, -t6 bj5 - t6 bj4 bk,  
-t7 bj6 - t7 bj5 bk, -t8 bj7 - t8 bj6 bk, -t9 bj8 - t9 bj7 bk, -t10 bj9 - t10 bj8 bk, -t11 bj10 - t11 bj9 bk,  
-t12 bj11 - t12 bj10 bk, -t13 bj12 - t13 bj11 bk, -t14 bj13 - t14 bj12 bk, -t15 bj14 - t15 bj13 bk }
```

```
FindSequenceFunction[
```

```
{-t2 bj - t2 bk, -t3 bj2 - t3 bj bk, -t4 bj3 - t4 bj2 bk, -t5 bj4 - t5 bj3 bk, -t6 bj5 - t6 bj4 bk,  
-t7 bj6 - t7 bj5 bk, -t8 bj7 - t8 bj6 bk, -t9 bj8 - t9 bj7 bk, -t10 bj9 - t10 bj8 bk, -t11 bj10 - t11 bj9 bk,  
-t12 bj11 - t12 bj10 bk, -t13 bj12 - t13 bj11 bk, -t14 bj13 - t14 bj12 bk, -t15 bj14 - t15 bj13 bk }
```

```
-  $\frac{t (t b_j)^{\#1} (b_j + b_k)}{b_j}$  &
```