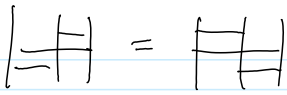
OneCo-150424

April-24-15 9:44 AM



Cheat Sheet OneCo

http://drorbn.net/AcademicPensieve/2015-04/
initiated 14/4/15; modified 23/4/15, 11:35am

Background. $\delta e^\gamma = e^\gamma \cdot \left(\delta\gamma /\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma}\right) = \left(\delta\gamma /\!/ \frac{e^{\operatorname{ad}\gamma}-1}{\operatorname{ad}\gamma}\right) \cdot e^\gamma$

The differential of $\gamma = \operatorname{bch}(\alpha,\beta)$:

$$\delta\gamma /\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} = \left(\delta\alpha /\!/ \frac{1-e^{-\operatorname{ad}\alpha}}{\operatorname{ad}\alpha} /\!/ e^{-\operatorname{ad}\beta}\right) + \left(\delta\beta /\!/ \frac{1-e^{-\operatorname{ad}\beta}}{\operatorname{ad}\beta}\right)$$

Models. • In $[x,y] = \delta x$, $xf(y) = f(y+\delta)x$. If $\delta^2 = 0$, $[x, f(y)] = \delta f'(y)x$.
• In $[x,y] = \delta x + z^2$, $xf(y) = f(y+\delta)x + \frac{z^2}{\delta}(f(y+\delta) - f(y))$. If $\delta^2 = 0$, $[x, f(y)] = \delta f'(y)x + z^2 f'(y)$.
• If $S_n := \sum_{k=0}^{n-1} A^k C B^{n-1-k}$ then $AS_n - S_nB = A^nC - CB^n$ so $S_n = (L_a - R_B)^{-1}(A^nC - CB^n)$.

Deriving Gassner. $\mathcal{L}^{2Dw}$ is $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo locality, $[a_{ij}, a_{ik}] = 0$, $[a_{ik}, a_{jk}] = -[a_{ij}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, and $[a_{ij}, a_{ji}] = b_i a_{ji} - b_j a_{ij}$. Acts on $V = \mathbb{Q}[\![b_i]\!]\langle x_i = a_{i\infty}\rangle$ by $[a_{ij}, x_i] = 0$, $[a_{ij}, x_j] = b_i x_j - b_j x_i$. Hence $e^{\operatorname{ad} a_{ij}} x_i = x_i$, $e^{\operatorname{ad} a_{ij}} x_j = e^{b_i} x_j + \frac{b_j}{b_i}(1 - e^{b_i}) x_i$. Renaming $y_i = x_i/b_i$, $t_i = e^{b_i}$, get $[e^{\operatorname{ad} a_{ij}}]_{y_i, y_j} = \begin{pmatrix} 1 & 1-t_i \\ 0 & t_i \end{pmatrix}$.

The $\mathcal{L}^{2Dw}$ Adjoint representation. $e^{t \operatorname{ad} a_{ij}}$ acts by

$$a_{kl} \mapsto a_{kl}, \qquad a_{ik} \mapsto a_{ik}, \qquad a_{kj} \mapsto e^{-tb_i} a_{kj} + \frac{b_k}{b_i}(1 - e^{-tb_i}) a_{ij},$$

$$\text{and} \quad a_{jk} \mapsto e^{tb_i} a_{jk} + \frac{b_j}{b_i}(1 - e^{tb_i}) a_{ik} \qquad \text{also for } k = i.$$

Implementation/verification: pensieve://2015-04/nb/ZeroCo.pdf.
Interpretation? "TT -Artin" ?

2Dv. b: bracket trace; c: cobracket trace; $\langle b, c\rangle = \delta \in \{0,1\}$; $\deg b_i = \deg c_j = \deg a_{ij} = \deg \delta = 1$.
$\mathcal{A}^{2Dv}$ is $\mathbb{Q}[\![\delta]\!]FA(b_i, c_j, a_{ij})$ (so $\mathcal{L}^v = \{f + f^{ij} a_{ij}\}$) modulo locality, $[a_{ij}, a_{ik}] = c_k a_{ij} - c_j a_{ik}$, $[a_{ik}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, $[a_{ij}, a_{jk}] = (c_j - b_j) a_{ik} + b_i a_{jk} - c_k a_{ij}$, $[a_{ij}, a_{ji}] = ?$, $[a_{ij}, b_i] = -[a_{ij}, b_j] = -[a_{ij}, c_i] = [a_{ij}, c_j] = \delta a_{ij} - b_i c_j$, $[b_i, c_j] = 0$.

$$a_{ij} f = f^\delta a_{ij} - \frac{b_i c_j}{\delta}(f^\delta - f), \quad [a_{ij}, f] = (f^\delta - f)\left(a_{ij} - \frac{b_i c_j}{\delta}\right),$$

with $f^\delta := f /\!/ \begin{pmatrix} b_i \to b_i + \delta & b_j \to b_j - \delta \\ c_i \to c_i - \delta & c_j \to c_j + \delta \end{pmatrix}$.

The Ascending Algebra $\mathcal{A}_+^{2Dv}$. Same but with only a_{ij}, $i < j$.

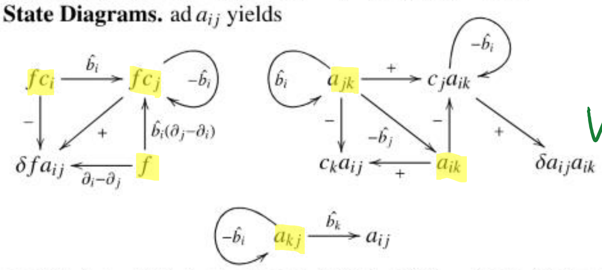
The primitivity condition. $\ker\left(f + f^{ij} a_{ij} \mapsto \delta f + f^{ij} b_i c_j\right)$. (Ignoring multiple arrows).

The OneCo Quotient is $\delta^2 = \delta c_i = c_j c_k = 0$, so

$$\mathcal{L}^{1co} = \Big\{(f + f^k c_k) + (f^{ij} + f^{ijk} c_k) a_{ij} + \delta f^{ijkl} a_{ij} a_{kl} : \\ f, f^{ij} \in \mathbb{Q}[\![\delta, b_i]\!]; f^i, f^{ijk}, f^{ijkl} \in \mathbb{Q}[\![b_i]\!]\Big\}.$$

Then $[a_{ij}, f + f^k c_k] = (\partial_i f - \partial_j f - f^i + f^j)(\delta a_{ij} - b_i c_j)$.

State Diagrams. $\operatorname{ad} a_{ij}$ yields



so with $\phi_0 := \phi(0)$, $\phi_1 := \phi_0'$, and $\phi_\downarrow(x) := (\phi(x) - \phi_0)/x$, $\phi(\operatorname{ad} a_{ij})$ is

$$\begin{aligned}
fc_i &\mapsto \phi_0 fc_i + (b_i\phi_{\downarrow\downarrow}(-b_i) - \phi_1)\delta f a_{ij} + b_i \phi_\downarrow(-b_i) fc_j \\
fc_j &\mapsto \phi(-b_i) fc_j + \phi_\downarrow(-b_i)\delta f a_{ij} \\
f &\mapsto \phi_0 f + b_i\phi_\downarrow(-b_i)(\partial_j f - \partial_i f)c_j \\
&\quad + (b_i\phi_{\downarrow\downarrow}(-b_i) - \phi_1)(\partial_j f - \partial_i f)\delta a_{ij} \\
a_{ik} &\mapsto \phi_0 a_{ik} + \phi_1 c_k a_{ij} - \phi_\downarrow(-b_i) c_j a_{ik} - \phi_{\downarrow\downarrow}(-b_i)\delta a_{ij} a_{ik} \\
a_{jk} &\mapsto \\
a_{kj} &\mapsto \\
a_{ij} &\mapsto a_{ij}
\end{aligned}$$

To do. • Perhaps I should find a way to highlight the fact that v is a perturbation of w.