

2Dv Relations

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(150409) 2D1co: b : bracket trace; c : cobracket trace $c_i c_j = 0$;
 $\langle b, c \rangle = \delta$. Relations: locality and $[a_{ij}, a_{ik}] = ?$, $[a_{ij}, a_{jk}] = ?$, $[a_{ik}, a_{jk}] = ?$, $[b_i, a_{ij}] = ?$, $[b_j, a_{ij}] = ?$, $[c_i, a_{ij}]$, $[c_j, a_{ij}] = ?$,
 $[b_i, c_i] = ?$. swap γ / β

$$[a_{ij}, a_{ik}] = \left| \begin{array}{c} \uparrow \\ \rightarrow \\ \downarrow \end{array} \right| = \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \downarrow \end{array} \right| = \gamma \left(\left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| - \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \leftarrow \end{array} \right| \right) \\ = \gamma (c_k a_{ij} - c_j a_{ik})$$

$$[a_{ik}, a_{jk}] = \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \downarrow \end{array} \right| = \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| = \beta \left(\left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| - \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \leftarrow \end{array} \right| \right) \\ = \beta (b_j a_{ik} - b_i a_{jk})$$

$$\delta_T: \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| \rightarrow [a_{ij}, a_{ik}] + [a_{ij}, a_{jk}] + [a_{ik}, a_{jk}] = 0$$

$$[a_{ij}, a_{jk}] = \left| \begin{array}{c} \uparrow \\ \rightarrow \\ \downarrow \end{array} \right| = -[a_{ij}, a_{ik}] - [a_{ik}, a_{jk}] = \\ = -\gamma (c_k a_{ij} - c_j a_{ik}) - \beta (b_j a_{ik} - b_i a_{jk}) \\ = (\gamma c_j - \beta b_j) a_{ik} + \beta b_i a_{jk} - \gamma c_k a_{ij}$$

$$[b_i, a_{ij}] = \left| \begin{array}{c} \uparrow \\ \rightarrow \\ \downarrow \end{array} \right| = \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \downarrow \end{array} \right| = \gamma \left(\left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| - \delta \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| \right) \\ = \gamma (b_i c_j - \delta a_{ij})$$

$$[b_j, a_{ij}] = \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \downarrow \end{array} \right| = \text{using } \left| \begin{array}{c} \uparrow \\ \rightarrow \\ \downarrow \end{array} \right| \begin{array}{l} a_{ik} \rightarrow b_i \\ a_{jk} \rightarrow b_j \\ c_k \rightarrow \delta \end{array} \\ = -(\gamma c_j - \beta b_j) b_i - \beta b_i b_j + \gamma \delta a_{ij} \\ = \gamma (\delta a_{ij} - b_i c_j) = \gamma \left(\delta \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| - \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right| \right)$$

$$[c_i, a_{ij}] = \left| \begin{array}{c} \rightarrow \\ \rightarrow \\ \downarrow \end{array} \right| = \text{using } \left| \begin{array}{c} \uparrow \\ \rightarrow \\ \downarrow \end{array} \right| \begin{array}{l} a_{ik} \rightarrow c_k \\ b_i \rightarrow \delta \end{array} / j \rightarrow i$$

$$= (\cancel{\gamma c_j} - \beta b_j) c_k + \beta \delta a_{jk} - \cancel{\gamma c_k c_j} \quad / \quad \begin{matrix} a_{ij} \rightarrow c_j \\ k \rightarrow j \end{matrix}$$

$$= \beta \delta a_{jk} - \beta b_j c_k \quad / \quad \begin{matrix} j \rightarrow i \\ k \rightarrow i \end{matrix}$$

$$= \beta (\delta a_{ij} - b_i c_j) = \beta (\delta | \rightarrow | - | \rightarrow^b c \rightarrow |)$$

$$[c_j, a_{ij}] = \begin{matrix} | \rightarrow | \\ \downarrow c \rightarrow | \\ | \rightarrow | \end{matrix} = \begin{matrix} | \rightarrow | \\ \downarrow \cancel{\gamma} \rightarrow | \\ | \rightarrow | \end{matrix} = \beta (| \rightarrow^b c \rightarrow | - \delta | \rightarrow |)$$

$$= \beta (b_i c_j - \delta a_{ij})$$

$$[b_i, c_j] = \begin{matrix} c \rightarrow | \\ \downarrow | \rightarrow b \\ | \rightarrow | \end{matrix} = - [c_j, a_{ij}] \quad / \quad \begin{matrix} a_{ij} \rightarrow b_j \\ c_j \rightarrow \delta \end{matrix}$$

$$= \beta (\delta b_j - b_j \delta) = 0.$$

In summary,

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Replacing $c \rightarrow c/\gamma$, $b \rightarrow b/\beta$, $\delta \rightarrow \frac{\delta}{\beta\gamma}$ we get:

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so it is:

(150409) 2Dv: b : bracket trace; c : cobracket trace; $\langle b, c \rangle = \delta$. Relations: locality, $[a_{ij}, a_{ik}] = c_k a_{ij} - c_j a_{ik}$, $[a_{ik}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, $[a_{ij}, a_{jk}] = (c_j - b_j) a_{ik} + b_i a_{jk} - c_k a_{ij}$, $[b_i, a_{ij}] = b_i c_j - \delta a_{ij}$, $[b_j, a_{ij}] = \delta a_{ij} - b_i c_j$, $[c_i, a_{ij}] = \delta a_{ij} - b_i c_j$, $[c_j, a_{ij}] = b_i c_j - \delta a_{ij}$, $[b_i, c_i] = 0$.

By scaling a_{ij}, b_j, c_j together, may assume $\delta \in \{0, 1\}$.

Missing so far: $[a_{12}, a_{21}]$:

