Cheat Sheet OneCo

http://drorbn.net/AcademicPensieve/2015-04/
initiated 14/4/15; modified 17/4/15, 9:18am

Background.

$d\,$exp: $$\delta e^\gamma = e^\gamma\cdot\left(\delta\gamma /\!\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma}\right) = \left(\delta\gamma /\!\!/ \frac{e^{\operatorname{ad}\gamma}-1}{\operatorname{ad}\gamma}\right)\cdot e^\gamma$$

The differential of $\gamma = \operatorname{bch}(\alpha,\beta)$:
$$\delta\gamma /\!\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} = \left(\delta\alpha /\!\!/ \frac{1-e^{-\operatorname{ad}\alpha}}{\operatorname{ad}\alpha} /\!\!/ e^{-\operatorname{ad}\beta}\right) + \left(\delta\beta /\!\!/ \frac{1-e^{-\operatorname{ad}\beta}}{\operatorname{ad}\beta}\right)$$

Models. • In $[x,y]=\delta x$, $xf(y)=f(y+\delta)x$. If $\delta^2=0$, $[x,f(y)]=\delta f'(y)$.

Deriving Gassner. Locality, $[a_{ij},a_{ik}]=0$, $[a_{ik},a_{jk}]=-[a_{ij},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, $[a_{ij},a_{ji}]=b_ia_{ji}-b_ja_{ij}$, b_i central. Acts on $V=\mathbb{Q}[\![b_i]\!]\langle x_i=a_{i\infty}\rangle$ by $[a_{ij},x_i]=0$, $[a_{ij},x_j]=b_ix_j-b_jx_i$. Hence $e^{\operatorname{ad}a_{ij}}x_i=x_i$, $e^{\operatorname{ad}a_{ij}}x_j=e^{b_i}x_j+\frac{b_j}{b_i}(1-e^{b_i})x_i$. Renaming $y_i=x_i/b_i$, $t_i=e^{b_i}$, get $[e^{\operatorname{ad}a_{ij}}]_{y_i,y_j}=\begin{pmatrix}1 & 1-t_i\\ 0 & t_i\end{pmatrix}$.

The $\mathcal{A}^w/2D$ Adjoint representation. $e^{\operatorname{ad}a_{ij}}$ acts by
$$a_{kl}\mapsto a_{kl},\qquad a_{ik}\mapsto a_{ik},\qquad a_{kj}\mapsto e^{-b_i}a_{kj}+\frac{b_k}{b_i}(1-e^{-b_i})a_{ij},$$
$$\text{and}\quad a_{jk}\mapsto e^{b_i}a_{jk}+\frac{b_j}{b_i}(1-e^{b_i})a_{ik}\qquad\text{also for } k=i.$$
An interpretation?

2Dv. b: bracket trace; c: cobracket trace; $\langle b,c\rangle=\delta\in\{0,1\}$. Relations: locality, $[a_{ij},a_{ik}]=c_ka_{ij}-c_ja_{ik}$, $[a_{ik},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, $[a_{ij},a_{jk}]=(c_j-b_j)a_{ik}+b_ia_{jk}-c_ka_{ij}$, $[a_{ij},a_{ji}]=?$, $[a_{ij},b_i]=-[a_{ij},b_j]=-[a_{ij},c_i]=[a_{ij},c_j]=\delta a_{ij}-b_ic_j$, $[b_i,c_i]=0$.
$$a_{ij}f=\left(f^\delta-\frac{b_ic_j}{\delta}(f^\delta-f)\right)a_{ij},\quad [a_{ij},f]=\left(1-\frac{b_ic_j}{\delta}\right)(f^\delta-f)a_{ij},$$
with $f^\delta := f /\!\!/ \begin{pmatrix} b_i\to b_i+\delta & b_j\to b_j-\delta\\ c_i\to c_i-\delta & c_j\to c_j+\delta\end{pmatrix}$.

The primitivity condition. $\ker\left(f+f^{ij}a_{ij}\mapsto \delta f+f^{ij}b_ic_j\right)$.

The OneCo Quotient is $\delta c_i=c_jc_k=0$. Then $[a_{ij},f]=(\delta-b_ic_j)\left[(\partial_{b_i}-\partial_{b_j}-\partial_{c_i}+\partial_{c_j})f\right]_{c_i=0}\cdot a_{ij}$.