Kapovitch's class, Mon Feb 2: Sisto on Agol's Theorem

February-02-15 11:11 AM

Thm Let M be a closed orientable hyperbolic 3-mfld. Then M is a finite cover that fibers over S^1.

"M virtually fibers"

Recall:

cover of M $\Longleftrightarrow$ subgroups of $\pi_1(M)$

finite sheeted $\Longleftrightarrow$ finite index.

Thm (Agol) M^3 as above, then M virtually fibers if $\pi_1 \hookrightarrow$ RAAG

Remark

surface

$$1 \to \pi_1(S) \to \pi_1(M) \to \mathbb{Z} \to 1$$

$$\pi_1(M) = \pi_1(S) \rtimes_\varphi \mathbb{Z}$$

$\Longrightarrow M$ fibers.

Γ: simplicial graph.

RAAG:

$$A_\Gamma = \langle v \in V\Gamma : [v,w]=1 \Leftrightarrow (v,w) \in E\Gamma \rangle$$

Empty graph: $\mathbb{Z}^V$

Complete graph: $FG(V)$

$\Gamma_1 \quad \Gamma_2 \; : \; A_{\Gamma_1} * A_{\Gamma_2}$

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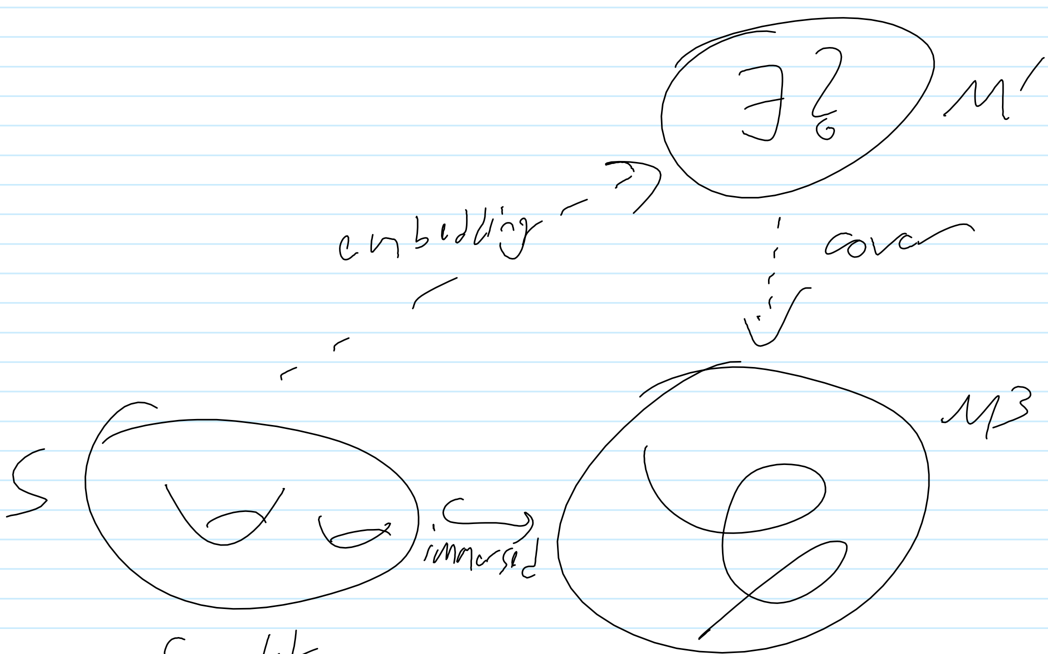
1. A_Γ embeds in SL_n so it is linear.
2. A_Γ is residually finite: for any $g \neq 1$ in A_Γ $\exists$ finite group F & $\varphi: A_\Gamma \longrightarrow F$ s.t. $\varphi(g) \neq 1$.
3. "Many" separable subgroups

Def $H < G$ is "separable" if $\forall g \notin H$ there is a finite-index $G' < G$ s.t. $H < G'$, $g \notin G'$. Equivalently, $\exists \varphi: G \longrightarrow \underset{\text{finite}}{F}$ s.t. $\varphi(g) \notin \varphi(H)$

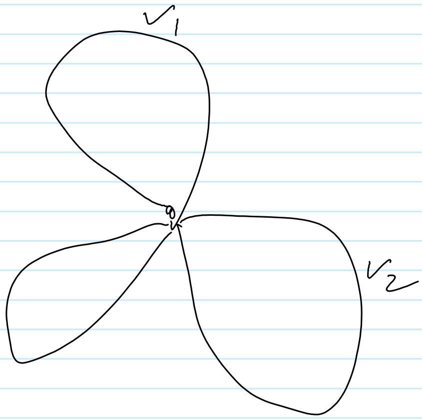


Scott

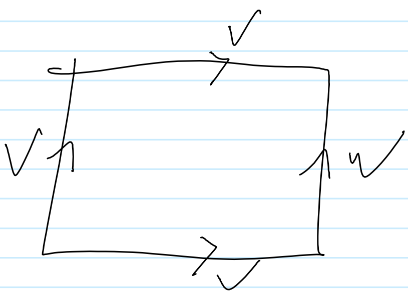
Thm $f: S \longrightarrow M^3$ be π_1-injective &

$f_*\pi_1(S)$ is seperable in $\pi_1(M)$, then there is an embedding $f': S \to M'$, M' a finite cover of M.

Let construct $K(A_\Gamma, 1) =: S_\Gamma$



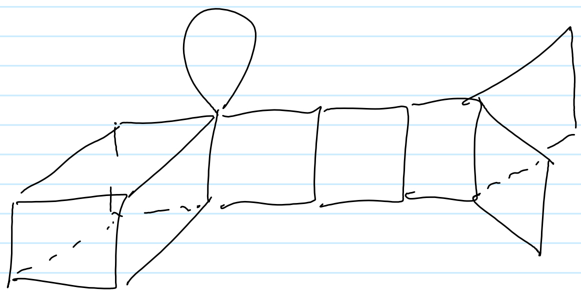
Add square for the commutators:



Add n-cubes for every n-complete subgraph of Γ.

S_Γ is the main example of a non-positively curved cube-complex.

A cube complex: a gluing of cubes along their faces:



etc.

<u>Def</u> a cube-complex is non-positively-curved if all links are flag

Flag:= if the 1-skeleton of a simplex is in, then so is the simplex.

HW: S_n is non-positively curved, hence aspherical.