

Pensieve header: Testing the expansion extension property for  $n$ -strand pure braid groups.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2015-02"];
<< ../Projects/WKO4/FreeLie.m
Conj[a_, b_] := (-b) ** a ** b;
FreeLie` implements / extends
{*, +, **, $SeriesShowDegree, <>, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, EulerE, Exp,
Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve,
Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ℓ, Λ, σ, ħ, −, −}.

SolvePB[n_] := SeriesSolve[
  Flatten@Table[
    Do[cs[n, i, j][1, k] = 0, {k, 1, n-1}, {l, k+1, n}]; cs[n, i, j][j, i] = 1;
    s[n, i, j] = DKS[n, cs[n, i, j]],
    {i, 1, n-1}, {j, i+1, n}
  ],
  And@@(Flatten@Table[
    Which[
      l < i || i < k < l < j, Conj[s[n, i, j], s[n, k, l]] ≡ s[n, i, j],
      l = i, Conj[s[n, i, j], s[n, k, l]] ≡ Conj[s[n, i, j], -s[n, k, j]],
      i = k && k < l < j, Conj[s[n, i, j], s[n, k, l]] ≡
        Conj[s[n, i, j], (-s[n, l, j]) ** (-s[n, i, j])],
      k < i < l < j, Conj[s[n, i, j], s[n, k, l]] ≡
        Conj[s[n, i, j], Conj[s[n, k, j], -s[n, l, j]] ** (-s[n, k, j])],
      True, BS@True
    ],
    {i, 1, n-1}, {j, i+1, n}, {k, 1, n-1}, {l, k+1, n}
  ])
]

SolvePB[2]

SolvePB[2]; s[2, 1, 2]
DKS[τ12, 0, 0, ...]

SolvePB[3]

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With[{n = 3}, Flatten@Table["s"[n, i, j] → s[n, i, j], {i, 1, n - 1}, {j, i + 1, n}]]
{s[3, 1, 2] →
  DKS[t12, 0, t13t13t23 cs[3, 1, 2][3, 1, 1, 2] + t13t23t23 cs[3, 1, 2][3, 1, 2, 2], ...],
s[3, 1, 3] → DKS[t13, 0, t13t13t23 cs[3, 1, 3][3, 1, 1, 2] +
  t13t23t23 cs[3, 1, 3][3, 1, 2, 2], ...], s[3, 2, 3] →
  DKS[t23, 0, t13t13t23 cs[3, 2, 3][3, 1, 1, 2] + t13t23t23 cs[3, 2, 3][3, 1, 2, 2], ...]}

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